An Extended Kalman Filter and an Appropriate Model for the Real-time Estimation of the Induction Motor Variables and Parameters

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ABSTRACT
This paper presents an efficient discrete-time second-order model of an induction motor for the rotor flux and parameters real-time estimation using an Extended Kalman Filter. Compared with the usual model, this model offers many advantages for real-time identification and fault diagnostic of the induction motor. Indeed not only, it remains stable and accurate for large sampling periods, but also it offers a better linearity with respect to the estimated parameters and it reduces the computational cost of the Kalman filter. Experimental results show the great accuracy and the fast convergence of the estimated parameters, even for sampling periods larger than 10 ms. In practice, the optimization of the measurement procedure and the realistic tuning of the Kalman filter has allowed the real evaluation of the parameters uncertainties and reveals the errors due to the model simplification of the motor such as the iron losses.

KEY WORDS
Extended Kalman Filtering, Real-time Identification, Fault Detection, Induction Motor

1. Introduction
These last decades were very fruitful for the induction motor control: we have assisted to the development of increasingly sophisticated control schemes, from simple scalar control methods to sensorless or auto-tuning control strategies. Now, performances are adequate for many industrial applications and research and development studies focusing on supervision and fault diagnosis. However, electric parameters characterizing the induction motor model can vary significantly during the normal operating point of the motor. This caused by phenomena such as; heating (stator and rotor), magnetic saturation or skin effect. Thus, the bad knowledge and the parameter variations can deteriorate the achieved control or supervision performances and decrease the output of the motor. Then, it may be necessary to estimate real-time parameters. An interesting feature of the Extended Kalman Filter (EKF) is its ability to estimate simultaneously the states and the parameters of a dynamic process. This is generally useful for both the control and the diagnosis of the process. However, an EKF is a rather complex algorithm and most of the previously published studies are not applicable for on-line parameter estimation. Hence, their computational load is very high or they require very short sampling times [1, 2, 3, 5, 7, 11]. However, reduced-order models of the induction motor have been proposed in previous papers [1, 2, 4]. These models were still relatively complicated. In this paper, we show how a convenient model can simultaneously reduce the computational load of an EKF and allows the use of large sampling periods without losing the estimation accuracy.

In practice, precautions must be taken to avoid inaccurate parameter estimation. Indeed, the measurement procedure should be optimized to reduce the eventual errors and noises. In [13], we have shown that some parameters are inaccurately estimated in the presence of important noises on the measurement, especially on the measured stator current. In this case, low-pass filters using small cut-off frequencies must be applied to the measured signals. In addition, the on-line estimation of the parameters must be supervised. Thus, we can estimate one or more parameter according to the measured signals dynamic behaviours [13].

The paper is organized as follows: section 2 reviews the fourth-order model and highlights its disadvantages for the parameter estimation. The second-order model and the EKF based on this model are presented in sections 3 and 4. The estimation accuracy using experimental data is discussed in section 5.

2. Fourth-order Model
The usual model of the induction motor is the two-phase model deduced from the Park transform. This model considers the stator voltages \( U_4 = [U_{sd} U_{sq}]^T \) as input and the stator currents \( Y_4 = [I_{sd} I_{sq}]^T \) as output. To simplify the observation equation, the stator currents are often chosen as state variables. The state vector is completed with the rotor flux components: \( X_4 = [I_{sd} I_{sq} \Phi_{rd} \Phi_{rq}]^T \). With
the usual hypothesis of symmetry and linearity (saturation, skin effect and core losses are neglected), the fourth-order model, in the mechanical frame, is given by the following equations:

\[ \begin{align*}
\dot{X}_4(t) &= A_4(\omega_m)X_4(t) + B_4U_4(t) \\
Y_4(t) &= C_4X_4(t)
\end{align*} \tag{1} \]

\[ A_4 = \begin{bmatrix} -\omega_m(t) & b\rho_r & b\omega_m(t) \\
R_r & 0 & 0 \\
0 & -\rho_r & -\rho_r \\
0 & R_r & 0 \end{bmatrix} \]

\[ B_4 = \begin{bmatrix} b & 0 & 0 & 0 \\
0 & b & 0 & 0 \end{bmatrix}^T 
\]

\[ C_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \end{bmatrix} \]

\[ a = \frac{R_s + R_r}{L_{fs}} , b = \frac{1}{L_{fs}} \text{ and } \rho_r = \frac{R_r}{L_r} \]

Where \( \omega_m = p\Omega_m \), \( \Omega_m \) is the rotor velocity and \( 2p \) is the pole number. The parameters \( R_s, L_{fs}, R_r, \) and \( L_r \) are, respectively, the stator resistance, the global leakage inductance \( (L_{fs} = \sigma L_s) \), the rotor resistance and the rotor inductance (all referred to the stator).

This model has many disadvantages for state or parameter estimation. First, because two state components (the stator currents) of this fourth-order model are accurately measurable, full-order observers deduced from this model are needlessly complicated. Second, the model contains simultaneously fast and slow modes: the current dynamics is far higher than the rotor flux dynamics. Moreover, the poles of this model vary with the mechanical speed and two of them are very lightly damped for high speeds. Therefore, great caution should be exercised to determine a stable and accurate fourth-order discrete-time model. Most authors use Euler approximation and a very small sampling period: 0.1 ms or less \([1, 2, 3, 8, 12]\). To allow larger sampling periods (up to 1 ms), more complex discretization schemes like the second-order series expansion of the matrix exponential are employed \([7, 9, 11]\). However, because the parameter sensitivity is approximately proportional to the sampling period, it is more advisable to avoid small sampling period: by improving the parameter identifiability, the risk of numerical divergence is reduced. A high sampling frequency may also be required to follow the dynamics of the fast input signals (the stator voltages). Lastly, the EKF uses a linearized model with respect to the estimated parameters. Due to the model complexity, this linearization is very cumbersome and involves heavy computations, especially when the second-order series expansion of the matrix exponential is employed. In conclusion, an EKF based on the usual fourth-order model implies a small sampling period and induces a severe computation volume and complexity.

### 3. Second-order Model

To get a simple model for rotor flux and parameter estimation, we consider the stator currents as input: \( U = [I_{sd}, I_{sq}]^T \) and the stator voltages as output: \( Y = [U_{sd}, U_{sq}]^T \). Then, the state vector only consists of the rotor flux components: \( X = [\Phi_{r d}, \Phi_{r q}]^T \). The mechanical frame is chosen as a reference frame for the electrical signals for two reasons. First, in that case the matrices of the state equation are constant and second, in steady state the frequency of the electrical variables is very small. With these assumptions, we obtain:

\[ \begin{align*}
\dot{X} &= AX + BU \\
Y &= C(\omega_m)X + D(\omega_m)U + E\dot{U} \tag{3} \\
A &= -\rho_r I, B = R_r I, C = -\rho_r I + \omega_m J, \\
D &= (R_s + R_r)I + L_{fs}\omega_m I, E = L_{fs}I \\
I &= \begin{bmatrix} 1 & 0 \\
0 & 1 \end{bmatrix} \quad \text{and} \quad J = \begin{bmatrix} 0 & -1 \\
1 & 0 \end{bmatrix} 
\end{align*} \tag{4} \]

The fact that the state vector only consists of the rotor flux offers a double advantage. First, the reduction of the state dimension reduces the computational volume and complexity. Second, thanks to its low dynamic, the rotor flux can be easily estimated: the second-order model has only real and slow poles. This property allows choosing a large sampling period \( T_e \), without failing the stability and the accuracy of the discrete-time model.

The discrete-time model is obtained by integrating the state equation (3) over the sampling period \( T_e \):

\[ X_{k+1} = e^{AT_e}X_k + \int_0^{T_e} e^{A(T_e-\tau)}BU(t_k + \tau)d\tau \tag{5} \]

Several solutions can be proposed to compute the integral in (5). This one depends on the input dynamic behaviour over the interval \([t_k, t_{k+1}]\). If the input vector \( U(t) \) is constant during the sampling period, that is if \( U(t) = U_k \) for \( t_k \leq t \leq t_k + 1 \), an exact discrete-time model can be determined:

\[ X_{k+1} = A_dX_k + B_dU_k \tag{6} \]

Because the state matrix \( A \) of the second-order model is diagonal, the discrete-time model can easily be defined analytically with respect to the electric parameters:

\[ A_d = e^{-\rho_r T_e}I \quad \text{and} \quad B_d = L_r(1 - e^{-\rho_r T_e})I \tag{7} \]

This discrete-time model can also be employed when the inputs are not constant; if their dynamics are low over the sampling period. For this reason, it is more interesting to consider the stator currents than the stator voltages as model inputs. However, to allow large sampling periods (several ms or more than 10 ms), the dynamics of the inputs (the stator currents) must be taken into account. In this case, we consider the interpolated value of the input over the period \( T_e \) \((U_k \text{ in (6) is replaced by } 0.5(U_k + U_{k+1}))\). For several tenth of ms more and for more precision in this case, the input components in (6) can be replaced by their mean values computed from the samples taken over
the period $T_e$ [13]. Therefore, the usual definition of the discrete-time matrices (7) can be kept. But, instead of using $U_k$, the samples of the input components, we use $\hat{U}_k$, their mean values over the period $T_e$:

$$X_{k+1} = A_dX_k + B_d\hat{U}_k$$

(8)

$\hat{U}_k$ is obtained by oversampling the input with a small period $t_e$ and using a trapezoidal approximation of the integration over $t_e$:

$$\hat{U}_k = \frac{1}{2N_e} \sum_{j=0}^{N_e-1} U(t_k + j t_e) + U(t_k + (j + 1) t_e)$$

(9)

Where $N_e = \frac{T_e}{t_e}$ is the oversampling factor. The period $t_e$ is chosen according to the dynamics of the stator currents. To simplify the controller implementation, $t_e$ can be the sample period of the current loop (if any) or the inverter modulation period.

The numerical computation of the derivative of the stator current is the unique difficulty in the output equation of the second-order model. It’s obtained by applying the bilinear transformation to a low-pass filtered derivative. To preserve the data coherency and avoid aliasing phenomenon, the same low-pass filter is applied to all signals. To reduce the bilinear transformation error, the derivative is computed with the short sampling period $t_e$.

4. Rotor Flux and Parameter Estimation by EKF

This paper is focused on the discrete-time motor model, however our objective is to estimate the motor parameters in real-time. In this section we highlight how the second-order model can simplify the implementation of the EKF for accurate estimations.

4.1 Stochastic Extended Model

To separately estimate the resistances, which may slowly vary with temperature, and the inductances, which depend on the saturation level, the parameter vector is chosen as:

$$\theta_k = [R_c(t_k) \quad L_{fs}(t_k) \quad R_r(t_k) \quad N_r(t_k)]^T$$

With $N_r$ is the inverse of the rotor inductance ($N_r = \frac{1}{L_r}$). To apply the EKF for simultaneous state and parameter estimation, one must extend the state vector by the parameter vector: $X_{e,k} = [X_k \quad \theta_k]^T$. The state extension implies a non-linear model. Moreover, the Kalman filter approach assumes that the deterministic model of the motor is disturbed by Gaussian and centered white-noise: the state noise and the measurement noise. We suppose that the parameters are constant or vary slowly ($\dot{\theta}(t) = 0$), the stochastic extended model is obtained by combining the model presented in (8) and the parameter dynamics affected by Gaussian and centered white-noise:

$$X_{e,k+1} = f(X_k, \theta_k) + W_{e,k}$$

(10)

$$Y_k = g(X_k, \theta_k) + V_k$$

(11)

With,

$$f = \left[ A_d(\theta_k)X_k + B_d(\theta_k)\hat{U}_k \right]_{\theta_k}$$

$$g(X_k, \theta_k) = C_k(\omega_m, \theta_k)X_k + D_{e,k}(\omega_m, \theta_k)U_k + E(\theta_k)\hat{U}_k$$

$W_{e,k} \text{ and } V_k$ are the extended state and measurement noises respectively. Their covariance matrices are:

$$Q_{e,k} = E\{W_{e,k}W_{e,k}^T\} \quad \text{and} \quad R_k = E\{V_kV_k^T\}$$

(12)

4.2 Discrete-time EKF

The EKF consists of two phases. First, the extended state is predicted according to the expected value of the extended model given in (10):

$$\hat{X}_{e,k+1|k} = f(\hat{X}_k|k, \hat{\theta}_k|k)$$

(13)

$$\hat{Y}_{k+1|k} = g(\hat{X}_{k+1|k}, \hat{\theta}_{k+1|k})$$

(14)

Then, this prediction is corrected by injecting the output estimation error:

$$\hat{X}_{e,k+1|k+1} = \hat{X}_{e,k+1|k} + K_{k+1}(\hat{Y}_{k+1} - \hat{Y}_{k+1|k})$$

(15)

The Kalman gain $K$ is deduced from the properties of the stochastic model to minimize the variance of the estimation error. However, because the Kalman filter is a linear observer, the extended model must be linearized with respect to the estimated extended state. Thanks to the simplicity of the discrete-time second-order model, this linearization can be computed very easily. Indeed, it’s very complicate when the fourth-order model is used. We have:

$$\begin{align*}
\hat{X}_{e,k+1|k} &= F_k \hat{X}_{e,k|k} + W_{e,k} \\
\hat{Y}_{k+1|k} &= G_{k+1} \hat{X}_{e,k+1|k} + V_{k+1}
\end{align*}$$

(16)

(17)

With $\hat{X}_e = X_e - \hat{X}_e$ the estimation error and with the following Jacobian matrices:

$$F_k = \left[ \frac{\partial f(X_k, \theta_k)}{\partial X_e} \right]_{X_e = \hat{X}_e} = \begin{bmatrix} A_d(\hat{\theta}_k) & J_X \\ 0 & I \end{bmatrix}$$

$$G_{k+1} = \left[ \frac{\partial g(X_{k+1}, \theta_{k+1})}{\partial X_e} \right]_{X_e = \hat{X}_e} = \begin{bmatrix} C_k & J_Y \end{bmatrix}$$

$$J_X = \left[ \frac{\partial X_{k+1,i}}{\partial \theta_j} \right]_{X_e = \hat{X}_e} \quad \text{and} \quad J_Y = \left[ \frac{\partial Y_{k+1,i}}{\partial \theta_j} \right]_{X_e = \hat{X}_e}$$

$i = 1, 2 \quad \text{and} \quad j = 1, 2, 3, 4$
With the additional assumption that there is no correlation between the system noises and state components, the optimal gain (the Kalman gain) and the covariance matrix of the estimation errors \( P_e = E\{X_e X_e^T\} \) are given by:

\[
P_{e,k+1|k} = F_k P_{e,k|k} F_k^T + Q_{e,k}\]

\[
P_{Y,k+1|k} = G_{k+1} P_{e,k+1|k}^T G_{k+1}^T + R_{k+1}\]

\[
K_{k+1} = P_{e,k+1|k}^T G_{k+1}^T P_{Y,k+1|k}^{-1}\]

\[
P_{e,k+1|k+1} = P_{e,k+1|k} - K_{k+1} P_{Y,k+1|k} K_{k+1}^T\]  

(18) \[ \tag{18} \]

(19) \[ \tag{19} \]

(20) \[ \tag{20} \]

(21) \[ \tag{21} \]

4.3 Tuning of the Covariance Matrices

The covariance matrices \( Q_{e,k} \) and \( R_k \) characterize the noises \( W_{e,k} \) and \( V_k \) respectively, which take into account of the model approximations and the measurement errors. Some authors consider these stochastic properties as free tuning parameters of the Kalman filter and use an LQG tuning approach [10]. We prefer to use a physical approach to obtain a more realistic tuning that permits to roughly evaluate the parameter uncertainties. However, a fine-tuning analysis based on measured data is used here.

The two first components of the extended state noise \( W_e \) are related to the rotor flux. For simplicity, we suppose that the two-phase components of this error are uncorrelated and share the same spectral density \( \sigma_\Phi^2 \). The four last components of the extended state noise characterize the parameter variations: they permit to tune the dynamics of the parameter estimation. Non-diagonal terms define the parameter couplings, for instance, the thermal coupling of \( R_s \) and \( R_o \) or the magnetic coupling of \( L_{fs} \) and \( L_r \). Therefore and for simplicity, the matrix \( Q_{e,k} \) is supposed to be diagonal:

\[
Q_{e,k} = \text{diag}(\sigma_\Phi^2, \sigma_\Phi^2, \sigma_{R_s}^2, \sigma_{L_{fs}}^2, \sigma_{R_o}^2, \sigma_{L_r}^2, \sigma_N^2)\]  

(22) \[ \tag{22} \]

The observation equation (4) is rather complicated and several noise origins should be considered. However, in practice, the main sources of error are the voltage measurements. By assuming uncorrelated two-phase errors of power spectral density \( \sigma_U^2 \), we get:

\[
R_k = \text{diag}(\sigma_U^2, \sigma_U^2)\]  

(23) \[ \tag{23} \]

Thus, the proposed analysis is based on an accurate simulation of the induction motor model considering the same conditions of the test presented on the figure 1. For every sampling period, the evaluation of variances \( \sigma_\Phi^2 \) and \( \sigma_U^2 \) is based on the spectrale density of the difference between two simulations: a simulation without noises and an other with noises. The experimental noises are introduced to the simulated data as correct as possible. For more details, one can refered to [13]. The values of these variances are summarized in Table 1.

### 5. Experimental Results

In this section, we illustrate the efficiency of our approach with experimental data obtained with a 2.2 kW induction motor. Measured signals (two components of the three-phase stator quantities and the rotor mechanical velocity) are presented in figure 1. Measurements of the stator voltages and currents were made by Hall-effect sensors and the rotor position was measured by an incremental encoder with a resolution of 2000 points. Because of the low frequency modulation of the static converter, efficient anti-aliasing filters were required: 5th order Bessel filters with a cut-off frequency of 1000 Hz. The data sampling period is \( t_e = 0.5 \text{ ms} \). Notice that this data sensitisizes all the parametres by controlling the motor by an important torque with different levels.

![Three-phase stator currents (A)](image)

<table>
<thead>
<tr>
<th>Discret-time period</th>
<th>( \sigma_\Phi^2 )</th>
<th>( \sigma_U^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_e = 1 \text{ ms} )</td>
<td>( 2.10^{-8} )</td>
<td>1.5</td>
</tr>
<tr>
<td>( T_e = 10 \text{ ms} )</td>
<td>( 20.10^{-8} )</td>
<td>0.15</td>
</tr>
<tr>
<td>( T_e = 40 \text{ ms} )</td>
<td>( 80.10^{-8} )</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 1. State and measurement noise variances

The figure 2 compares estimation results for three sampling periods: 1, 10 and 40 ms. Electric parameters were initialized with a significant error (about 50%) to show the fast and regular convergence of the estimates. It can be seen that the three sets of final estimates are very close. The fact that we allow the variation on the estimated parameters, by tuning the EKF via the parameter components of the covariance matrix \( Q_{e,k} \), the continuous-time model errors affect the estimations, especially for the rotor resistance \( R_o \) as shown in figure 3. Thus, it seems that the EKF estimates a global impedance of the motor, which characterised by four parameters only in our case. In reality, the simplification introduced to obtain the continuous-time model causes the variations revealed on the estimated parameters (see figure 3), especially in the presence of iron losses.

The parameter mean values, summarized in Table 2, show that only the inductance \( L_{fs} \) is sensible to the sam-
Figure 2. Estimated parameters; $T_e = 1,\ 10,\ 40\ ms$

Figure 3. Estimated parameters; $T_e = 1\ ms$

Figure 4. On-line estimated parameters; $T_e = 0.5\ ms$

6. Conclusion

We have shown in this paper that the second-order model and the extended Kalman filter are adapted to the tracking of the rotor flux and the electrical parameters perfectly. It’s possible to use sampling period of 10 ms or larger without losing the estimation precision. In practice, this allows the real-time implementation of an efficient parameter and rotor flux estimator without requiring a powerful processor.
References


