

# A New Modelling Of DC Machines Taking Into Account Commutation Effects

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**Abstract-** In some specific manufactured electromechanical applications, power electronic converters must be avoided due to hostile environment. In such a context, stand-alone DC machines are good candidates because of their high reliability and low cost. However, the mechanical commutation is a big problem in this kind of machine especially in high current and low voltage applications. Losses due to arching contacts can be significant. The authors present an original technique to take into account every electrical aspect of brushes contacts (mechanical sliding contact and arching contact). The aim of this study is to determine currents and voltages of conductors during commutation in order to reduce significantly losses due to sliding contact. These evolutions are very important data for Finite Element Analysis too.

**Keywords:** DC machines, Commutation, Modelling Technique, Brushes, Arching Contact

## I. INTRODUCTION

This paper deals with DC motors modelling taking into account commutation aspects. Indeed, the particular behaviour of sliding contacts between brushes and commutator can strongly influence DC motor characteristics such as output mechanical power, brushes life, commutator temperature, and durability of machines...

Nowadays, Finite Element (FE) Analysis is commonly used in order to optimize electrical drives. The aim of this numerical approach is to find optimal parameters (geometric dimensions, magnetic characteristics...) in order to reach performances defined in a specification sheet. Contrary to synchronous machines with sinusoidal current supply, it is hard to define current distribution in DC machines taking into account commutation. This repartition is a compulsory data for FE Analysis. Moreover, it is difficult to define experimentally this repartition especially in high currents and low voltages applications where commutation has significant effects.

In the following paper, we consider that there is a contact between brush and segment when electrical current circulates between them. This contact can be mechanical or arching contact.

This paper demonstrates with a simple algorithm how the sliding contacts between brushes and commutator can

influence the current distribution in the armature windings of DC motors. Voltage drops due to mechanical sliding contacts or arching contacts will be taken into account.

Moreover, this algorithm gives us the full distribution of current in the conductors of the rotor for a given number of brushes and a specific configuration around the commutator. Furthermore, this type of algorithm, combined with a FE software allows more faithful simulations in order to determine an optimal solution which minimises the commutation losses in brushes and by this way increases the durability of DC machines.

## II. THE MODELING PROCESS

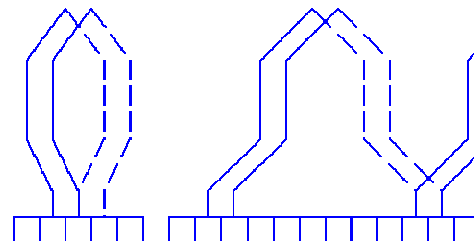


Fig. 1. Developed diagram for simple lap and simple wave winding

In any DC motor, the rotor is made of a specific periodic winding which can be simple, multiple, lap or wave winding (see Fig 1). A “section” is the elementary part of wire connected between two commutator segments. For each rotation angle, this closed topology of  $N_s$  sections is modified by the different connections made by the brushes. These connections can be mechanical sliding contacts or electrical arching contacts. Thus, a specific brush has two main functions: first, transmit electrical energy to rotating parts and secondly, bypass two or more consecutive commutator segments in order to commutate current into considered sections.

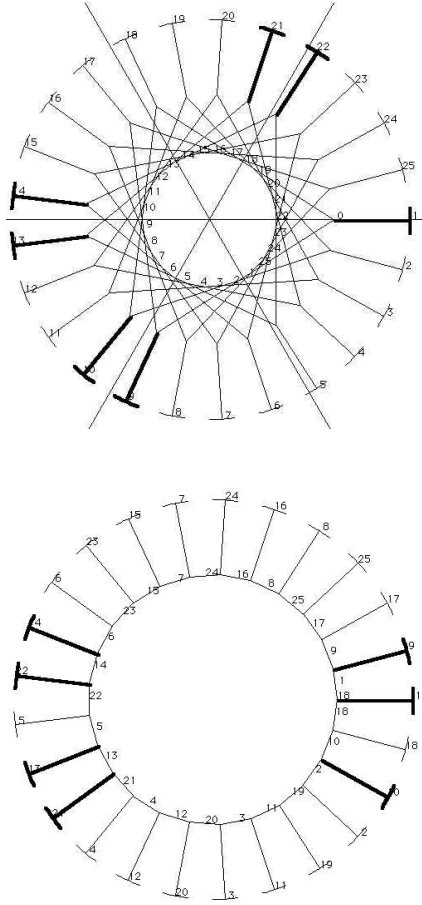


Fig. 2. Mechanical and electrical diagram of a DC machine

A typical machine is shown in Fig 2. Every brush contact is bolded. This machine has four brushes. In the second figure, we can see the two different current paths and the different number of sections in commutation under positive and negative brushes.

For every rotation angle, the full topology with  $N_s$  unknowns becomes a smaller equivalent topology with  $N_c$  unknowns. The algorithm is able to classify sections in view of brushes connections and simplify the initial topology with specific matrix formalism. Then, the  $N_c$  equations are solved and electrical characteristics such as currents and voltages of all the sections can be easily computed. From the evolution of these variables, we can evaluate the quality of the commutation process for a specific machine. Moreover, the algorithm draws up a balance sheet of all sources of losses in the machine.

### III. MATHEMATICAL FORMALISM

The armature winding of the machine is modelled by the following matrix equation:

$$U_{sec} = [R_{sec}] \cdot I_{sec} + [L_{sec}] \cdot (dI_{sec}/dt) + E_{sec} \quad (1)$$

Where:

$U_{sec}$  is the vector of the section voltages

$I_{sec}$  is the vector of the section currents

$E_{sec}$  is the vector of the section back-EMF

$R_{sec}$  and  $L_{sec}$  are square matrices ( $N_s \cdot N_s$ ) of section resistances and inductances (self and mutual)

Taking into account the succession of  $N_c$  contacts (mechanical or arching contacts) between brushes and collector, the algorithm sorts sections in  $N_c$  groups. Each group includes sections connected in series between two consecutive contacts.

Then, a matrix  $A$  named “connection matrix” is filled. This matrix has  $N_c$  rows and  $N_s$  columns and is full of “0” except in  $A_{i,j}$  when section  $j$  belong to group  $i$ . Especially in this case,  $A_{i,j}$  is equal to “1”.

This specific matrix helps us to perform the calculation of the initial topology because (1) becomes:

$$U_g = [R_g] \cdot I_g + [L_g] \cdot (dI_g/dt) + E_g \quad (2)$$

With

$$R_g = A \cdot R_{sec} \cdot A'$$

$$L_g = A \cdot L_{sec} \cdot A'$$

$$E_g = A \cdot E_{sec}$$

Where:

$U_g$  is the vector of the group voltages

$I_g$  is the vector of the group currents

$E_g$  is the vector of the group back-EMF

$R_g$  and  $L_g$  are matrices ( $N_c \cdot N_c$ ) of group resistances and inductances

With this reduced equation system, a standard Runge-Kutta solver performs the calculation of (2) and gives the current flowing in every group at the next time step.

$$I_{sec} = A' \cdot I_g \quad (3)$$

Finally, a matrix  $C$  named “inverse transformation matrix” is required. This matrix has  $N_c$  rows and  $N_s$  columns and is equal to  $A$  except that there is only one “1” in any rows. Considering the next topology, this matrix

allows us to gather sections together at the next time step with the following equation:

$$I_g = C \cdot I_{sec} \tag{4}$$

We have to be cautious with regard to equal currents section in this equation especially when arching contacts occur.

#### IV. THE SLIDING CHARACTERISTIC

In order to evaluate voltage drops of any contacts, a sliding characteristic has been designed. This curve represents for a given brush and a given rotor, the evolution of voltage drop as a function of the contact angle (defined in Fig 3) between brush and collector segment. When a segment is rotating and arrives under a brush, the contact angle increases until a maximum  $\theta_b$  (which is the minimum between the brush angle and the segment angle). Then, the contact angle is constant and decreases until zero. Finally, it becomes virtually negative until the arc extinction ( $\theta_a$ ). This evolution is shown in Fig 4.

If the contact angle between the brush and the considered segment is positive, there is mechanical contact and the voltage drop is small. It increases when the contact angle decreases to reach a maximum situated between 0.2 and 1V [1],[2],[3].

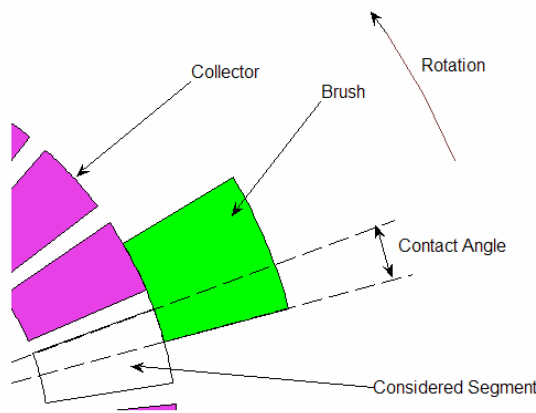


Fig. 3. Schematics of the contact angle between brush and collector segments

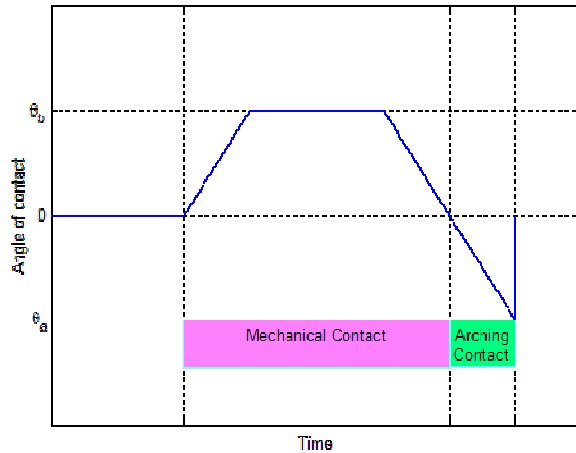


Fig. 4. Evolution of the Contact Angle between a brush and a segment

If the distance is negative, there is an arching contact and the voltage drop increases exponentially until the arc extinguishment.

This evolution of the voltage drop is exposed in Fig 5.

At every time step, the program determines the different connections and computes two vectors. These vectors named  $V_p$  and  $V_n$  (respectively for positive and negative brushes) include mechanical and arching voltage drops for all the connections. When the current of section in commutation reaches conduction current, the program extinguishes the arc.

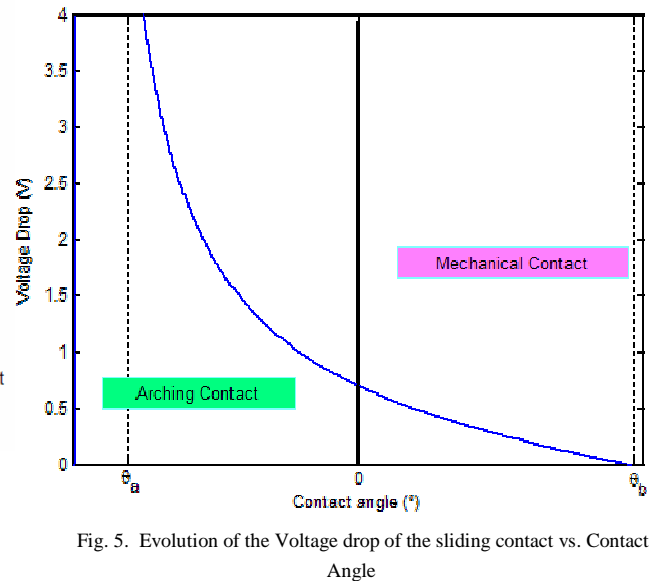


Fig. 5. Evolution of the Voltage drop of the sliding contact vs. Contact Angle

V. RESISTIVE AND INDUCTIVE MATRICES COMPUTATION

To solve the model, it is necessary to know self and mutual inductance and resistive matrices.

The resistive matrix is simply equal to:

$$R_{sec} = r \cdot Id_{[Ns,Ns]} \quad (5)$$

With  $r = 8,24 \cdot 10^{-4} \Omega$

Where:

r is the resistance of one insulated section

Id is the identity matrix (size Ns)

For the inductance matrix, a FE analysis is needed in order to determine self and mutual inductance coefficients.

With only one section supplied by a known current source, the magnetic flux in each section can be computed and self and mutual inductance coefficients are deduced. Geometry of the machine and evolution of coefficient are shown in Fig 6 and 7.

This method permits us to determine every coefficient of the first line of the matrix. The following lines are deduced by circular permutations as described in (6).

$$M_{1,1} = M_{2,2} = L_1 = L_2 = L_n \quad (6)$$

$$M_{1,2} = M_{2,3} = M_{3,4} = \dots\dots\dots$$

$$L_{sec} = \begin{bmatrix} L_1 & M_{1,2} & & & \\ M_{2,1} & L_2 & M_{2,3} & & \\ & & L_3 & & \\ & & & L_4 & \\ & & & & L_n \end{bmatrix}$$

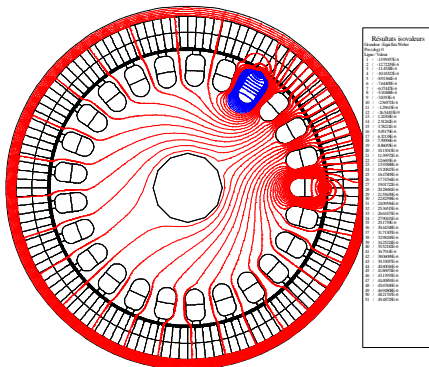


Fig. 6. Magnetic field path for a typical DC machine

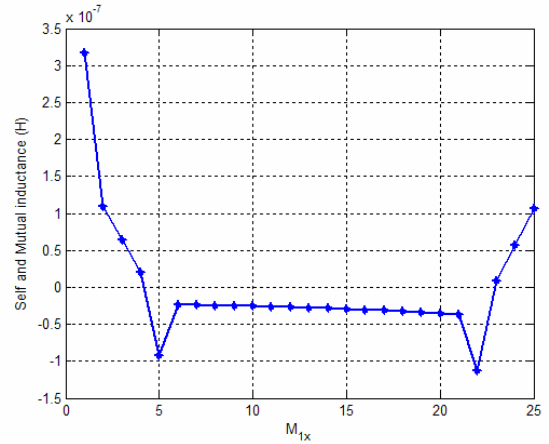


Fig. 7. Evolution of coefficients for the considered machine

If the machine has salient poles on the stator the program can manage the FE software in order to compute coefficients for every position of the rotor.

VI. CASE STUDY

For this section, the machine geometry shown above (Fig 6) will be studied. This machine has wave windings. The quality of the commutation will be evaluated when the number of brushes goes up from two to four. We will evaluate sources of losses for these two configurations. The different parameters of this machine are described in Tab 1.

In the first configuration, the winding is commonly supplied by two diametrically spaced brushes. In order to decrease the current in brushes and to enhance commutation quality, a configuration with four brushes is studied. For industrial applications, these new brushes (see Fig 8) are placed in such a way that positive and negative supply wires do not cross. This mounting arrangement increases compactness of the rear parts of the machine.

TABLE I  
MAIN CHARACTERISTICS OF THE MACHINE

Symbol	Value	Description
$N_p$	3	Number of Pole Pairs
$N_s$	25	Number of sections
$N_{seg}$	25	Number of collector segments
$W_p$	4	Winding pitch (in slot)
$C_p$	8	Collector pitch (in slot)
$\theta_{seg}$	11.3°	Angle of a collector segment
$\theta_{bal}$	17.2°	Angle of a brush
$\theta_p$	14.4°	Segment pitch

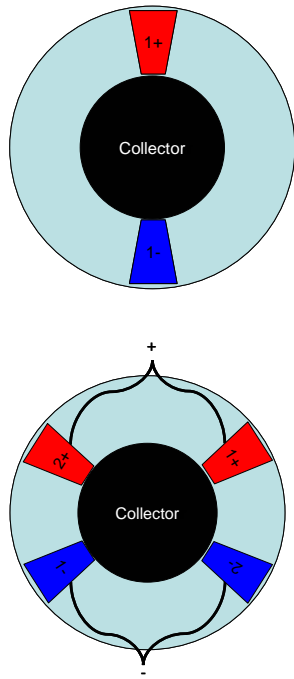


Fig. 8. Position of brushes in the two configurations

The inductor is made of a magnet where the induction along the air gap is deduced by the relation:

$$B_n = B_r \cdot \cos(Np.\theta) \quad (5)$$

*With  $B_r = 0.4T$*

For this kind of machine, the topology (see Fig 9) comes down to two groups of sections under positive and negative brushes and two groups of section representing current paths ( $U_{gN1+1}$  and  $U_{gN1+N2+2}$ ). The connection vectors ( $V_p$  and  $V_n$ ) include mechanical and arching voltage drops. At every time step equations of the topology are solved.

### VII. RESULTS

For each simulation, the program gives several curves (Fig 10 to 13). From the different averages of these curves, we can summarize the results (see Tab 2). For all sources of losses, the proportion of the power supply is mentioned into brackets.

From these simulations, we can conclude that two extra brushes increase mechanical contact losses but decrease

significantly (-6%) arching losses. Obviously, the total efficiency of the machine is increasing of 6%.

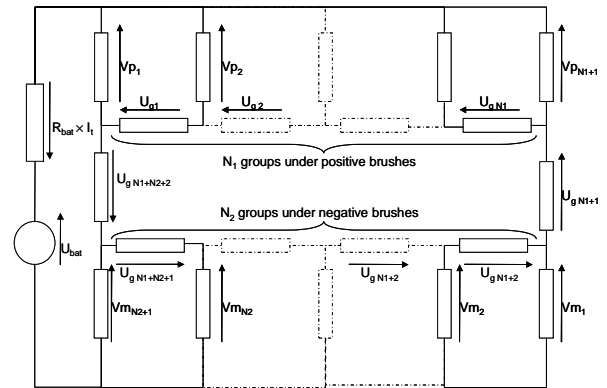


Fig. 9. General topology for considered machines

TABLE 2  
BALANCE SHEET OF THE TWO CONFIGURATIONS

	Configuration n°1 Two brushes	Configuration n°2 Four brushes
Supplied Power (W)	3452 W	3463 W
Arching Losses (W)	- 1278 W (37%)	- 1005 W (29%)
Mechanical Contact Losses (W)	-123 W (4%)	- 233 W (7%)
Joule Losses (W)	-1009 W (29%)	- 963 W (28%)
Electromagnetic Power (W)	1042 W (30%)	1262 W (36%)

The evolution curves of mechanical sliding and arching contact under positives brushes are shown on Fig 12 and Fig 13.

The program gives us the total repartition of the current in the rotor. For comparison, current curves under three consecutive poles are shown for configuration 1 and configuration 2 (see Fig 10 and 11).

In an ideal motor, the current has to commute from positive values to negative values between inductor poles (dash lines). In the two graphs, we can see easily that the current needs several mechanical degrees to switch. During this commutation time, conductors produce a antagonistic torque on the rotor which is completely disastrous. For this machine with a small number of large slots, the commutation phenomenon is very important. In fact, more than one third of the total number of conductors can create a resistive torque.

To solve this problem, manufacturer shift brushes of few degrees with respect to the inductors to the detriment of electromagnetic interaction between stator and rotor.

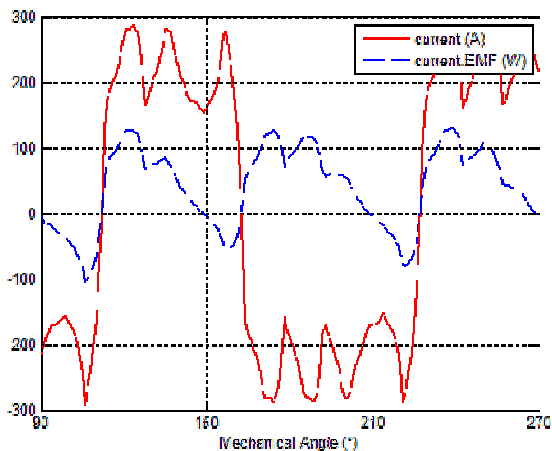


Fig. 10. Evolution of two variables for configuration 1

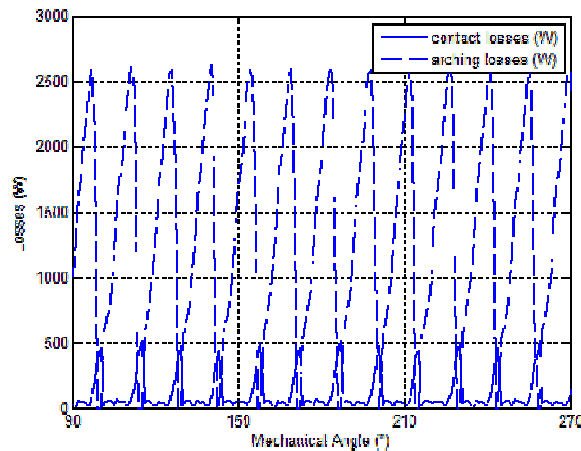


Fig. 12. Evolution of losses for configuration 1

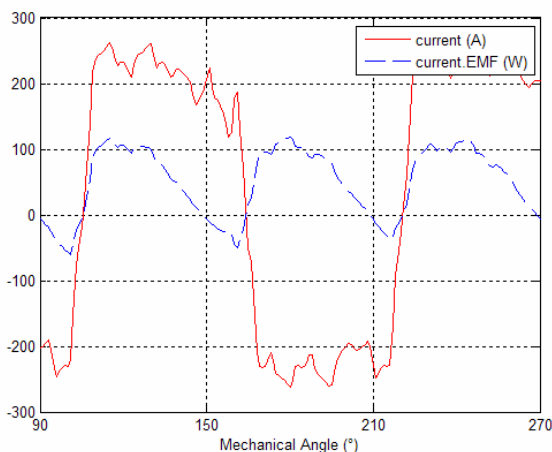


Fig. 11. Evolution of two variables for configuration 2

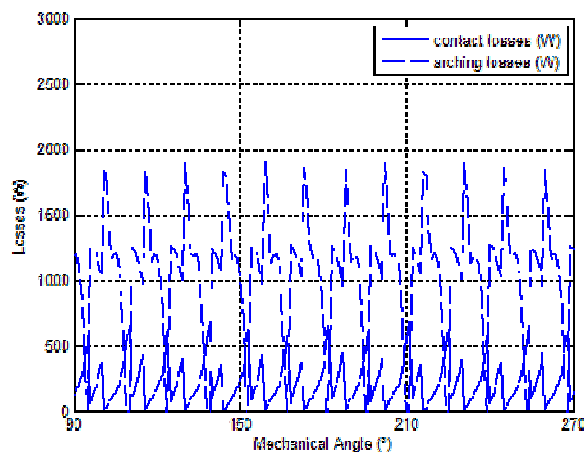


Fig.13. Evolution of losses for configuration 2

VIII. CONCLUSION

In this paper, a new technique of modeling DC machines has been exposed. This technique takes into account voltage drops due to mechanical sliding contacts and arching contacts which are very important for low voltage and high current applications.

In this type of machines, commutation is bad and takes enough time to create antagonist torque on the rotor. The authors present two configurations with two and four brushes and evaluate losses for these machines. The next step is to minimize arching losses and enhance commutation quality with a optimal number of brushes and a specific configuration.

IX. REFERENCES

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