A New Predictive Control Strategy Dedicated to Salient Pole Synchronous Machines

Nicolas Patin, Member IEEE
University of Technology of Compiègne,
Laboratoire d'Electromécanique de Compiègne,
Rue Personne de Roberval, BP 20525, 60205 Compiègne, France
Tel: +33 (0)3 44 23 79 66, fax: +33 (0)3 44 23 79 37
e-mail: nicolas.patin@utc.fr

Abstract-**This paper deals with a new predictive control strategy adapted to Salient Pole Synchronous Machines. Indeed, it is shown in this study that, in such a case, classical predictive control schemes require complex prediction calculations. So, they lead to complex implementations in a real-time context. Moreover, these techniques, which are well adapted to FPGA implementations due to their high parallelism capabilities, must be designed using IP building blocks as small as possible. First, αβ models of the Salient Poles Synchronous Machine (SPSM) and Voltage Switching Inverter (VSI) are presented. Then, principles of predictive control are given using an αβ reference frame. This method is directly applied to the SPSM and the complexity of the corresponding controller design is highlighted. A modified control scheme, using a rotating dq reference frame, is finally proposed and resulting changes on the controller design are discussed.**

Index Terms—**Salient Pole Synchronous Machine, VSI, Predictive Control, Implementation, FPGA, Real-Time processing, Parallelism**

I. INTRODUCTION

Speed drive controller strategies have been extensively investigated in last decades. However, most of the proposed solutions presented in the literature have been primarily designed using a sequential control algorithm. Thus, the obtained performances can be limited by this imposed controller's architecture. More recently, alternatives have been studied using FPGA based solutions. In such a context, new control schemes can be proposed because of specific FPGA parallel processing capabilities.

For instance, predictive control of electrical machines supplied by voltages switching inverters (VSI) is adapted to this kind of implementation. Indeed, VSI are controlled by three binary inputs (switching functions of each leg noted c_1, c_2 and c_3 respectively). Thus, the input vector $\mathbf{u}=[c_1,c_2,c_3]^t$ takes a value which belongs to a finite set S_u (containing 2^3=8 elements). This kind of system allows us to propose a control strategy which is based on the prediction of state vector $\mathbf{x}$ (of the controlled machine) trajectories for each case. Finally, the controller selects a value of the input vector among all elements of S_u on the basis of a given criterion (*e.g.* minimization of the norm of the error between the predicted state vector $\mathbf{x}^\#$ and the state vector reference $\mathbf{x}^{ref}$).

Parallel processing is usable in such control strategies. So, FPGA are good candidates for implementation of this kind of controller. Performances are thus dramatically increased in comparison with those obtained with a DSP implementation of similar control schemes: an important step in the design process consists in the choice of an adapted target for a given algorithm: DSP controllers and FPGA have both advantages and drawbacks which lead to use them for separate and specific tasks in which they give respectively the best performances.

However, even if classical predictive control strategies seem to be interesting in terms of robustness and simplicity, they require an accurate model in order to bring optimal control performances. Such a solution has been presented in [3] for the control of a synchronous machine (SM) and in [1] for a doubly fed induction machine (DFIM) used as a variable speed constant frequency generator. However, in [1], the SM model used for the armature currents prediction is limited to smooth pole synchronous machines. In section II, a model of salient poles synchronous machines is given. The $\alpha\beta$ model of VSI is also recalled. Predictive control principles used in [2]–[5] are then presented in section III and applied to the SPSM model. Results are then discussed in order to evaluate the complexity of the implementation of the obtained prediction algorithm. In section IV, a modified algorithm is proposed using a SPSM *dq* model. It is then shown that this rotating reference frame allows simplifying prediction equations implemented in the controllers but requires some additional operations that are also presented.

II. SYSTEM MODELING

A. Salient Pole Synchronous Machine

The model of the Salient Poles Synchronous Machine used in this study is based on the following hypotheses:

- Linearity of the magnetic behavior,

978-1-4244-1736-0/08/$25.00

- Geometrical symmetries (invariant machine by rotations of electrical angles equal to ±2π/3) of the bipolar equivalent machine (see Fig. 1),
- Identical electrical parameters for each armature winding,
- Sinusoidal variations of stator/rotor reluctances.

Notice that $p\theta$ is an electrical angle that allows us to study a generic bipolar machine without model changes due to the number of pole pairs. Please note that the angle θ corresponds to the actual mechanical position of the rotor in the stator reference frame.

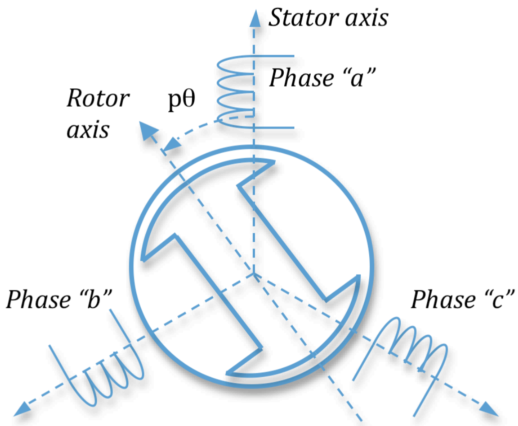

Fig. 1. Bipolar equivalent representation of a salent pole synchronous machine

The "*abc*" model presented below allows introducing all parameters of the machine:

$$(v_{3s}) = R_s.(i_{3s}) + \frac{d(\psi_{3s})}{dt} \tag{1}$$

and

$$(\psi_{3s}) = (L_{ss}(\theta)).(i_{3s}) + \Psi_{pm}.\begin{pmatrix} \cos(p\theta) \\ \cos\left(p\theta - \frac{2\pi}{3}\right) \\ \cos\left(p\theta + \frac{2\pi}{3}\right) \end{pmatrix} \tag{2}$$

where (v_{3s}), (i_{3s}) and (ψ_{3s}) are the armature windings voltages, currents and fluxes respectively. Then, several parameters are introduced in these equations:

- Armature windings resistance R_s
- Permanent magnet/Stator flux Ψ_{pm}
- Stator/Stator inductances matrix $(L_{ss}(\theta))$ defined as follows

$$(L_{ss}(\theta)) = \begin{pmatrix} L_{s0} & M_{s0} & M_{s0} \\ M_{s0} & L_{s0} & M_{s0} \\ M_{s0} & M_{s0} & L_{s0} \end{pmatrix}$$

$$+L_{s2}.\begin{pmatrix} \cos(2p\theta) & \cos\left(2p\theta + \frac{2\pi}{3}\right) & \cos\left(2p\theta - \frac{2\pi}{3}\right) \\ \cos\left(2p\theta + \frac{2\pi}{3}\right) & \cos\left(2p\theta - \frac{2\pi}{3}\right) & \cos(2p\theta) \\ \cos\left(2p\theta - \frac{2\pi}{3}\right) & \cos(2p\theta) & \cos\left(2p\theta + \frac{2\pi}{3}\right) \end{pmatrix} \tag{3}$$

where L_{s0}, M_{s0} and L_{s2} are three other parameters of the synchronous machine. Notice that this model can be reduced for a smooth pole machine by taking $L_{s2} = 0$.

Then, "*abc*" to $\alpha\beta$ (*i.e.* without rotation) transformation is applied to the SPSM equations (1) and (2). It gives

$$(v_{2s}) = R_s.(i_{2s}) + \frac{d(\psi_{2s})}{dt} \tag{4}$$

with

$$(\psi_{2s}) = L_{cs0}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \rho_s.P(p\theta)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.P(-p\theta)\right].(i_{2s})$$
$$+\Psi_{pm}.P(p\theta).\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

where L_{cs0} is the cyclic inductance L_{s0}-M_{s0}, ρ_s=3L_{s2}/(2.L_{cs0}) is the saliency ratio of the machine and $P(.)$ is the 2x2 rotation matrix defined as follows

$$P(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \tag{6}$$

B. *Voltage Switching Inverter*

The voltage switching inverter supplying the machine is supposed to be connected to an ideal DC link (*i.e.* a voltage source noted V_{dc}) as it is shown in Fig. 2. All v_{iM} ($i = a$, b or c) voltages can be easily expressed as a function of V_{dc} and switching functions f_i defined as follows:

- $f_i = 1$ if K_i is turned on and K'_i is turned off
- $f_i = 0$ if K_i is turned off and K'_i is turned on

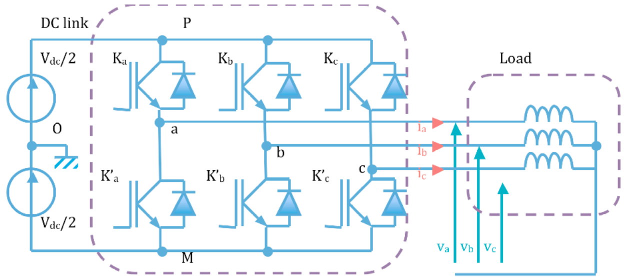

Fig. 1. Three-phase voltage switching inverter

Thus, it gives

$$v_{iM} = V_{dc}.f_i \tag{7}$$

The modeling of the inverter requires expressing voltages applied to the load. It can be seen that composed voltages can be directly given but, simple voltages v_i expressions cannot be established. Indeed, a complementary assumption about the load (*i.e.* the synchronous machine) is required: the zero sequence component of the simple voltages is supposed to be equal to zero:

$$v_a + v_b + v_c = 0 \tag{8}$$

And it gives

$$(v_{3s}) = \frac{V_{dc}}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}\begin{pmatrix} f_a \\ f_b \\ f_c \end{pmatrix} = V_{dc}.(G_{VSI}).\begin{pmatrix} f_a \\ f_b \\ f_c \end{pmatrix} \tag{9}$$

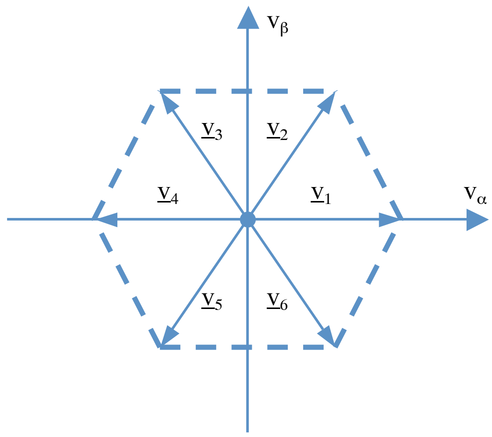

Fig. 3. Constellation of instantaneous αβ VSI output voltages

Finally, "*abc*" to $\alpha\beta$ transformation can be applied to this expression. It can be noticed that (G_{VSI}) can be expressed as a function of the "*abc*" to $\alpha\beta$ transformation matrix C_{32}:

$$C_{32}=\begin{pmatrix}-1 & 0\\ -1/2 & \sqrt{3}/2\\ -1/2 & -\sqrt{3}/2\end{pmatrix},\ \text{So}\ (G_{VSI})=\frac{2}{3}C_{32}.C_{32}^{t}$$

Thus

$$(v_{2s})=\frac{2}{3}V_{dc}.C_{32}{}^{t}\begin{pmatrix}f_a\\ f_b\\ f_c\end{pmatrix} \tag{10}$$

Then, using this expression, a constellation of instantaneous voltages can be established (see Fig. 3)[1]. It can be seen that 7 voltage vectors are available (6 active vectors + 1 null vector). It can be also noted that the number of vectors is inferior to the dimension of S_u indicated in the introduction. Indeed, two input combinations ($\mathbf{u}$=(0 0 0)^t and $\mathbf{u}$=(1 1 1)^t) correspond to the same null vector. Thus, the dimension of S_u can be reduced: dim(S_u)=7.

III. Predictive Control

C. Principles

On the basis of the models established in the previous sections, a control strategy can be proposed, taking into account the discrete behavior of the inverter. Indeed, the control input value belonging to a finite set (dim(S_u)=7). Thus, by using a differential equation of the load current in a prediction block included in the controller, currents variations (during a sampling period T_s) can be predicted for each voltage vector available at the VSI output.

A generic predictive control routine can be described as follows:

1. **Routine "Predictive_Control(i_{2s}^{ref})"**
2. *Begin*
3. Currents references i_{2s}^{ref} specified by the main program
4. Measurement of the actual load currents
5. *For each voltage vector V_i (0<i<6)*
   Prediction of the current $i_{2s}^{\#}(i)$
6. *End For*
7. Choice of the voltage V_i minimizing a given criterion (e.g. error between $i_{2s}^{\#}$(i) and i_{2s}^{ref})
8. *End*

The key point of this algorithm is the prediction of the current i_{s2} for a given VSI output voltage V_i (noted $i_{2s}^{\#}$).

If a generic electrical machine is considered, its armature windings can be described by the following electrical equation:

$$(v_{2s})=R_s(i_{2s})+L_s\frac{d(i_{2s})}{dt}+(e_{2s}) \tag{11}$$

Using this equation, a prediction equation, based on a discrete derivation, can be established:

$$\begin{aligned}&\left(i_{2s}{}^{\#}[n+1,k]\right)=\left(i_{2s}{}^{meas}[n]\right)\\&+\frac{T_s}{L_s}\left((V_k)-R_s\left(i_{2s}{}^{meas}[n]\right)-\left(\hat{e}_{2s}[n]\right)\right)\end{aligned} \tag{12}$$

where

- $\left(i_{2s}{}^{\#}[n+1,k]\right)$ is the predicted load current vector at the next (n+1) sampling instant for a given VSI output voltage vector (V_k),
- $\left(i_{2s}{}^{meas}[n]\right)$ is the actual (measured) load current vector at the present (n) sampling instant,
- (V_k) is the VSI output voltage vector n°k,
- $(\hat{e}_{2s}[n])$ is the estimated back-e.m.f. of the load (machine) at the present (n) sampling instant.

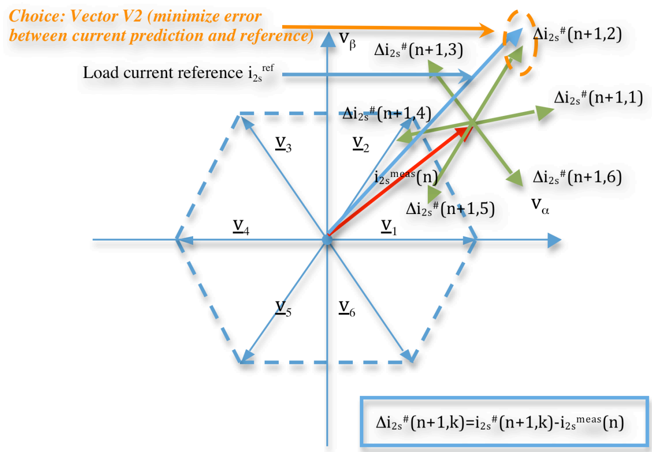

Fig. 4. Predictions of load currents variations for each instantaneous VSI output voltage vector.

This technique leads to a very simple implementation giving good results in terms of reference tracking and robustness against load parameters uncertainties. However, if this generic algorithm is applied to salient poles synchronous machine: predictions require several complex operations that dramatically increase execution times as it is shown in the next subsection.

[1] All vectors in this constellation are associated to complex voltages noted $\underline{V}_i$. These complex voltages correspond in the text to 2 dimensional vectors noted (V_{2s}).

D. SPSM Application

A prediction equation can be derivate from (4)–(5) using the same method that for (11). Thus, it gives:

$$\begin{aligned}&\left(i_{2s}^{\#}[n+1,k]\right)=\left(i_{2s}^{meas}[n]\right)\\&+T_s\left(L_{\alpha\beta}(\theta)\right)^{-1}\left(\left(V_k\right)-R_s\left(i_{2s}^{meas}[n]\right)-\left(E_{2s}[n]\right)\right)\end{aligned}\tag{13}$$

where

$$\left(L_{\alpha\beta}(\theta)\right)=L_{cs0}\left(\begin{pmatrix}1&0\\0&1\end{pmatrix}+\rho_s.P(p\theta)\begin{pmatrix}1&0\\0&-1\end{pmatrix}P(-p\theta)\right)\tag{14}$$

and

$$\left(E_{2s}[n]\right)=p\Omega.\begin{pmatrix}\Psi_{pm}\begin{pmatrix}\cos(p\theta)\\\sin(p\theta)\end{pmatrix}\\+\dfrac{3L_{s2}}{2}.P(p\theta)\begin{pmatrix}0&1\\1&0\end{pmatrix}P(-p\theta)\left(i_{2s}^{meas}[n]\right)\end{pmatrix}\tag{15}$$

These equations show that prediction requires several multiplications of constant and variables quantities with sine and cosine functions of the electrical angle $p\theta$ (due to $P(p\theta)$ and $P(-p\theta)$) of vector (i_{s2}^{meas}). That is an important issue for real-time processing. The avoidance of such operations could simplify the implementation of predictive control schemes on reasonably powerful targets (DSP or FPGA controller boards).

It can be noticed that the classical solution is based on predictions in a stationary reference frame in which VSI output voltage vectors are constant whereas load current are sinusoidal in steady state operation. Within this framework, the prediction algorithm yields to few simple operations. However, the situation is widely different with salient pole machines because of their anisotropic behavior (*d* and *q* axes).

If predictions are performed in a *dq* (rotor synchronous) reference frame, the equation (13) is simplified as follows

$$\begin{aligned}&\left(i_{dq}^{\#}[n+1,k]\right)=\left(i_{dq}^{meas}[n]\right)\\&+T_s\left(L_{dq}\right)^{-1}\left(\left(V_{dq}\right)-R_s\left(i_{dq}^{meas}[n]\right)-\left(E_{dq}[n]\right)\right)\end{aligned}\tag{16}$$

with

$$\left(L_{dq}\right)=\begin{pmatrix}L_d&0\\0&L_q\end{pmatrix}\tag{17}$$

where $L_d=(1+\rho_s).L_{cs0}$ and $L_q=(1-\rho_s).L_{cs0}$

and

$$\left(L_{dq}\right)=\begin{pmatrix}L_d&0\\0&L_q\end{pmatrix}\tag{17}$$

$$\left(E_{dq}[n]\right)=p\Omega.\begin{pmatrix}\Psi_{pm}\begin{pmatrix}1\\0\end{pmatrix}\\+\begin{pmatrix}L_d&0\\0&L_q\end{pmatrix}\left(i_{dq}^{meas}[n]\right)\end{pmatrix}\tag{18}$$

In this equation, a rotation operation has to be applied to the load current vector before all predictions calculations. This calculation is as complex as the one required for the back-e.m.f. calculation indicated in (15). Then it must be noticed that the inversion of (L_{dq}), needed in (16), is very simple because it is a diagonal and constant matrix, whereas the inversion of ($L_{\alpha\beta}(\theta)$) required in (13) implies much calculations due to 4 variables coefficients to determinate at each sampling period. Even if this inversion can be performed at the beginning of the process (before all predictions calculations), it is simpler in the *dq* reference frame than in the αβ one. Then, inverted matrices haven to be multiplied to a given vector determined for each prediction: (L_{dq}) being diagonal, this calculation is also simpler for the *dq* reference frame solution.

However, using this solution, voltage vectors provided to the load by the inverter are not constant any more. Indeed, rotations introduced by the reference frame rotation must be applied to them, as it is illustrated in Fig. 5. These operations reduce the interest of the choice of a rotating *dq* reference frame but as it is indicated in the next section, symmetries of the initial VSI output voltage constellation allow simplifying this preliminary task.

Thus, all these remarks lead us to conclude that predictions should be executed in the *dq* reference frame in order to obtain the simplest controller, for both DSP and FPGA-based implementations.

IV. ROTATING CONSTELLATION AND IMPLEMENTATION CONSEQUENCES

In a stationary reference frame, the instantaneous VSI output voltages form the constellation presented in Fig. 3. The “abc” to “*dq*” transformation introduces a rotation that gives a new moving constellation, as shown in Fig. 5.

Thus, a real-time calculation of all voltage vectors must be performed before load currents predictions. As it is indicated in the previous section, constellation’s symmetries can be exploited in order to simplify calculations. Indeed, each vector transformation result can be used for the symmetrical one because they always satisfy the following relationships:

$$\begin{cases}\underline{V}_4=-\underline{V}_1\\\underline{V}_5=-\underline{V}_2\\\underline{V}_5=-\underline{V}_3\end{cases}\tag{15}$$

Moreover, in the proposed controller, V_{dc} is supposed to be strictly constant. Thus, VSI output voltage vectors calculations are as reduced as possible. In a real-time context, trigonometric functions are usually implemented using a table in memory.

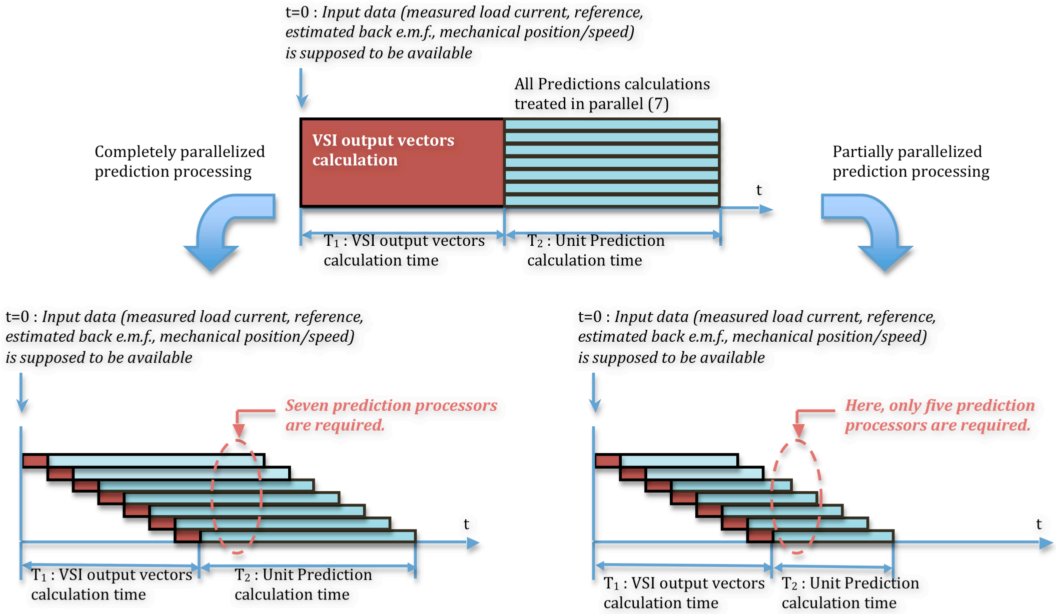

Fig. 7. Completely or partially parallelized predictions

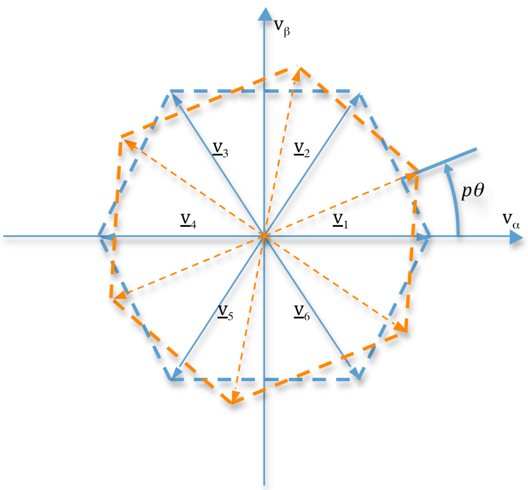

Fig. 5. Rotating constellation of VSI output voltages (*dq* reference frame)

Thus, calculations delays are reduced to read access times. Using offsets corresponding to instantaneous voltages vectors angles in addressing data, only one table is required. Six values must be read successively in the table and stocked in registers for all 12 vectors components. However, it must be noticed that two of these values are also required for "*abc*" to "*dq*" transformations applied to the load current measurements[1]. But then, predictions calculations, executable in parallel, can be achieved in a very reduced time due to (16)–(18) expressions in comparison with (13)–(15).

[1] Load currents references are supposed to be already available in the rotating dq reference frame.

V. Algorithm Sequence and Parallelizable Tasks

As it is indicated since the introduction, the main interest of FPGA-based implementation is the parallel computations capability of such circuits. In the proposed control strategies, some preliminary calculations are required before predictions but then, the predictions calculations can be performed in parallel as it is shown in Fig. 6.

Indeed, it can be seen in Equation (16) that only two quantities are common to all predictions:

- Measured dq load current
- Estimated back-e.m.f. of the load

Obviously, each *dq* VSI output voltage vector must be calculated before corresponding prediction calculation. Assuming that measured load current and estimated back-e.m.f. are already available, prediction calculation n°*k* can be performed if and only if voltage vector (V_k) is also available.

Now, a time-optimal architecture can be synthesized on the basis of the knowledge of the VSI output voltage calculation execution time T_1 and prediction execution time T_2. Parallel processing mechanisms are illustrated by timings given in Fig. 7.

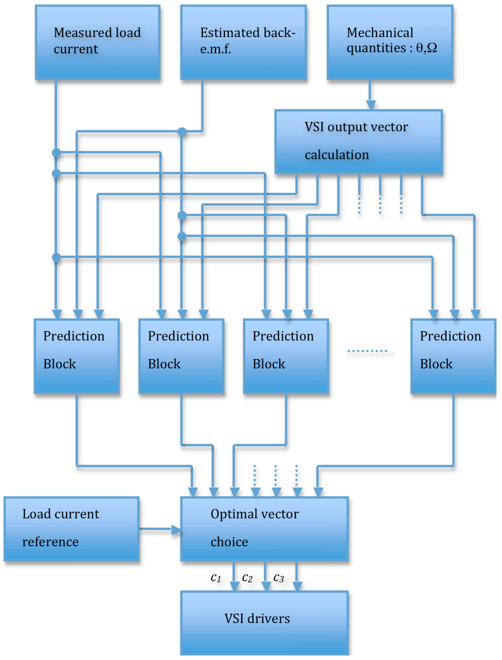

Fig. 6. Parallelized predictions calculations

VI. SIMULATIONS

The proposed controller has been validated by simulations using a *dq* model of a salient pole synchronous machine as it is shown in Fig. 7. All parameters of this simulation are given in Table I.

TABLE I
SIMULATION PARAMETERS

Parameter	Value	Unit
Armature resistance	10	Ω
d-axis inductance	458	mH
q-axis inductance	229	mH
Field	6	mWb
Simulation step	5	µs
Prediction sampling period	100	µs
DC bus voltage	560	V

The controller allows regulating very accurately *dq*-armature currents with a high dynamics. Even if this control strategy is sensitive to parametric variations, it seems to be quite robust against modeling uncertainties because sampling period is small enough to avoid large errors during quick reference variations but this subject should be further investigated.

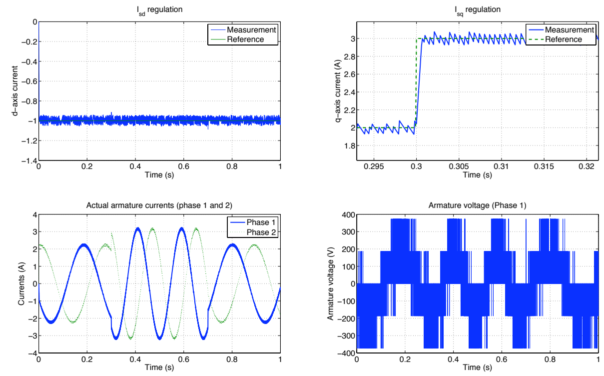

Fig. 7. Simulation results

VII. CONCLUSION

The proposed control scheme in a rotating reference frame greatly simplifies implementation of predictive strategies for salient pole synchronous machines. Even if this technique is particularly adapted to FPGA-based targets, it keeps a significant interest using it in DSP controller board. Indeed, in such a context, a classical solution established in a fixed $\alpha\beta$ reference frame would bring poor performances in comparison with theoretical potential of predictive methods only because of prohibitive execution time.

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