Comparison of optimization algorithms for the design of a brushless DC electric machine with travel time minimization

T.D. Nguyen, V. Lanfranchi, C. Doc and J.P. Vilain
University of Technology of Compiègne /Electromechanical Laboratory of Compiègne (LEC)
Compiègne, France

***Abstract*—This paper presents DC brushless (DCBL) machine design methods using the three Matlab's optimization algorithms. Instead of optimizing the machine's design with respect to a classic criterion such as the minimization of losses, of weight or cost (usually at one operating point), our criterion will be the time it takes for the machine to drive a variable load to fulfill its trajectory. This will require the calculation of the machine's behavior on the whole of its operating diagram (torque/speed diagram). A simplified analytical model of a DCBL machine is used in order to compare the performance of the three algorithms. The characteristics of the optimization results and the use of different algorithms in cascade will be discussed.**

I. Introduction

In the literature, many articles have proposed different methods to optimize the design of electrical machines based on one operating point (torque/speed) ([1], [2], [3], [4]). In [4] an optimization method using the sequence of genetic and gradient-based algorithms has been proven to give the optimal solution with a small calculation time. This method is applied for the design of a DCBL motor where the criterion is the total losses at only one particular operating point.

One particularity of our problem is that the machine has to drive a variable load, which is in function of the rotor's angular position (see Fig. 1). At first the load increases linearly, then it stays at the maximal position. The second particularity is our optimization criterion: the travel time. This "travel time" is defined as the time it takes for the machine's load to fulfill its trajectory. This method is different from an optimization method at one operating point.

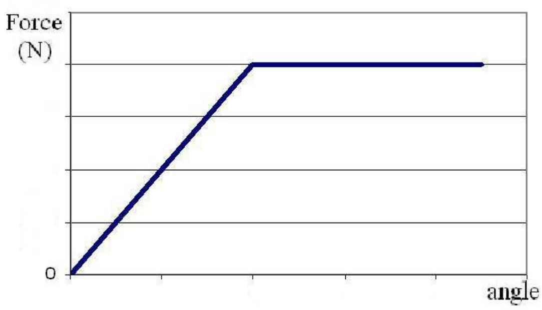


Figure 1. Evolution of the load with respect to the rotor's angular position

In this article, a simplified analytical model of a DCBL machine as well as a discretized travel time calculation method will be shortly presented, then the three optimization algorithms of Matlab (SQP, Direct Search and Genetic Algorithm) along with their advantages and drawbacks will be described. Finally the results obtained by different methods will be mentioned.

The DCBL machine in this article is a small one with the following fixed external dimensions: 70mm of length and 18mm of the outside radius.

II. Analytical model of a DCBL machine

Synchronous surface permanent magnet machines are famous for their high compacity and their high massic torque. In this section, a linear DCBL machine's analytical model (with a realistic flux density limit for the considered section) is presented. This model is validated and fine-tuned with help of FEM method (FLUX2D software).

A. Electromagnetic model

The main idea is to calculate the torque with torque constant from design parameters. In this analytical model, the torque constant K is assumed to be equal to the EMF constant. An equivalent LRE electric circuit (Fig.2) is needed to calculate the behavior of the motor. The EMF is calculated by the following formula [5]:

$$E = K\omega \qquad (1)$$

where ω is the machine's rotation speed.

The torque constant of a peripheral permanent magnet machine can be calculated by the following formula:

$$K = 4pNB_g R_a L_{st} N_{enc} \qquad (2)$$

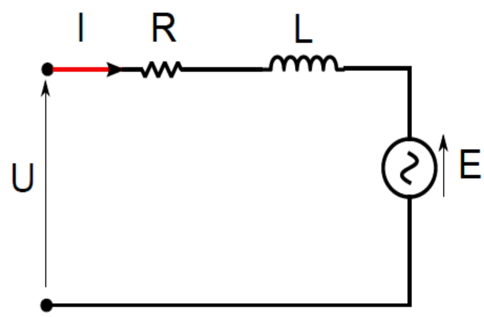


Figure 2. Simplified electric model of the machine

where

- p: number of pole pairs
- N: number of conductors per slot
- Nenc: number of slots per pole per phase
- Bg: flux density in the air gap
- Ra: average radius of the air gap
- Lst: machine's length

The flux density in the air gap Bg is calculated with help of the magnetic field circulation. It takes into account the geometric dimensions, the remanent flux density of permanent magnet and the saturated magnetic field.

$$B_g = f(geometric\ dimensions, H_{sat,} Br_T) \tag{3}$$

In order to be able to simulate the motor's behavior, we need to determine its inductance and resistance.

1. Coil inductance calculation:

The net phase winding inductance in a machine is calculated by [5]:

$$L = L_g + L_s + L_e \tag{4}$$

where

- L_g is the air gap inductance:

$$L_g = \frac{2\pi\mu_0 L_{st} R_{ro}}{L_a + \frac{l_m}{\mu_R C_\phi}} N^2 \tag{5}$$

  R_{ro} is the rotor's radius, L_a is the air gap's width, l_m is the depth of permanent magnet and C_ϕ is the relative magnet's coverage.

- L_s is the slot leakage inductance

$$L_s = N_m (2N)^2 \left[\frac{\mu_0 d_s L_{st}}{3\omega_{sb}} + \frac{\mu_0 d_t L_{st}}{\frac{\omega_{so}\omega_{sb}}{2} + \frac{\mu_0 d_{sh} L_{st}}{\omega_{so}}} \right] \tag{6}$$

where N_m is the number of slots.

- L_e is the end turn inductance

$$L_e = \frac{N_m \mu_0 \tau_{cp} N^2}{2} ln\left(\frac{\tau_{cp}\sqrt{\pi}}{\sqrt{2A_s}}\right) \tag{7}$$

where As is the cross sectional area of the air gap, the other variables are presented in Fig.3.

2. Resistance calculation:

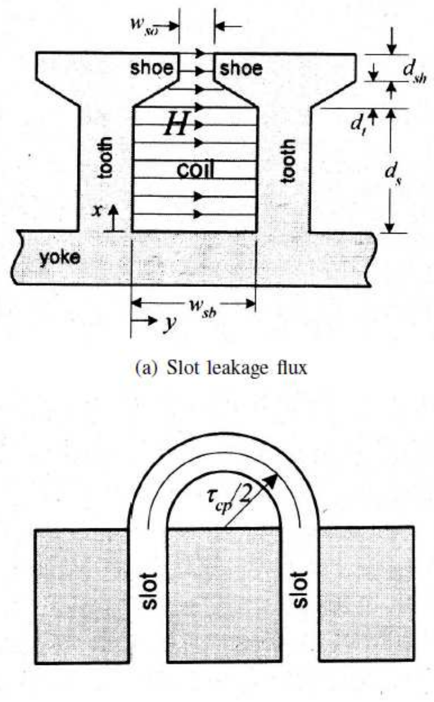
The total resistance of the motor is calculated from the total length of the conductor in the stator, this is a classic calculation and won't be presented here.

B. Mechanical calculation

Once the magnetic and the electric models have been made, the maximal torque can be calculated. From this maximal torque, the minimal radius of the drive shaft is calculated thanks to a material resistance formula, then the inertia of the whole rotor is calculated.

III. OBJECTIVE FUNCTION CALCULATION

In order to obtain the optimal solution for an optimization problem, one has to define an objective function along with its optimization variables and constraints if necessary. In this article, the objective function will calculate the travel time with three stages: acceleration, constant speed and acceleration. Because of



(a) Slot leakage flux

(b) End turn geometry approximation

Figure 3. Physical dimensions for calculating machine's inductance

the size of our machine, some dimensions have been fixed in order to satisfy dimensional requirements of the application. Table I presents some fixed and optimization variables. Among these variables, p and N_{enc} are discrete variables with the following constraints (due to the size of the machine):

- $1 \leq p \leq 3$
- $1 \leq N_{enc} \leq 2$

Unfortunately, Matlab's optimization algorithms cannot handle directly problems with discrete values, thus the optimization procedure is the following:

- Initializing the value of these two variables (6 possible combinations thanks to the above constraints).
- Proceeding the optimization with three rest variables using different algorithms available in Matlab.

In this article, only results with the initialization p = 3 and N_{enc} = 1 are detailed.

One can also notice that the variable N (number of conductors per slot) is also an integer value. The difference of this variable and the two above discrete variables is that N can vary in a large range of values. That's why we opted to do an optimization as if N were

Table I
TABLE OF VARIABLES OF THE DCBL MACHINE

L_{st}	machine's length	70mm
$R_{ext-stat}$	exterior radius of the stator	18mm
L_{is}	slot opening width	3mm
L_a	air gap's width	0.3mm
p	number of pole pairs	optim. variable (3)
N_{enc}	number of slot per pole per phase	optim. variable (1)
N	number of conductor per slot	optim. variable
l_m	permanent magnet's depth	optim. variable
$R_{int-stat}$	interior radius of the stator	optim. variable

continuous; the solution given by the optimizer for N will then be rounded to the closest integer value. Finally, the objective function value will be re-evaluated with the rounded value of N. This technique can lead to a problem called "integrality gap" which is that the optimal solution, with continuous variables, respects all the constraints whilst the neighbor points which correspond to discrete variables do not; and so the final solution, with discrete values, is far away from the solution given by the optimizer. In our case, no constraints are connected to N (cf. V), this technique is applicable.

The analytical model helps obtain the torque constant K, the inertia of the whole rotor J_{rotor}, the resistance R and inductance L of the machine. We will use these values to determine the dynamics (acceleration, speed and displacement).

Since we have a variable load, which depends on the angular position of the drive shaft, it is difficult to have a direct formula to calculate the travel time of the machine. We choose the method of discretizing the time into small intervals and calculate the displacement and speed of the machine in each interval. Figure 4 presents the calculation principle of the objective function. At the iteration 1, all values are set to be 0. Then at each iteration k, we execute the following calculations:

- The electromotive force of the motor:

$$EMF(k) = K\,\Omega\,(k-1) \tag{8}$$

- The current I which flows in the machine:

$$I(k) = \frac{U - EMF(k) + L\frac{I(k-1)}{dt}}{R + \frac{L}{dt}} \tag{9}$$

  where dt is time step.

- The resistive torque $Torque_{Res}(k)$ at each iteration is determined by Fig.1
- The Torque given by the motor at each iteration is calculated by

$$Torque(k) = K\,I(k) \tag{10}$$

- If the torque developed by the motor is superior to the resistive torque, the speed of the machine $\Omega\,(k)$ is calculated by

$$\Omega\,(k) = \Omega\,(k-1) + \frac{Torque(k) - Torque_{Res}(k)}{J_{rotor} + J_{load}}\,dt \tag{11}$$

where J_{rotor} and J_{load} are the inertia of the rotor and the load (included the tranmission system) seen by the motor. Otherwise, the travel time increases without the increase of displacement which will bring us to the limit time (f=0.4 s).

- The angular position is calculated by:

$$\theta(k) = \theta(k-1) + \Omega(k)dt \tag{12}$$

- And finally the travel time at iteration k is determined by

$$time(k) = time(k-1) + dt \tag{13}$$

At the end of each iteration, the angular position is compared with the final angular position of the motor, if the two values are equal, the calculation will stop and the value of the objective function is the value of time(k). Another comparison is conducted here to see if the travel time exceeds the limit time, if yes, the calculation is stopped, and the objective function is given the value 2,

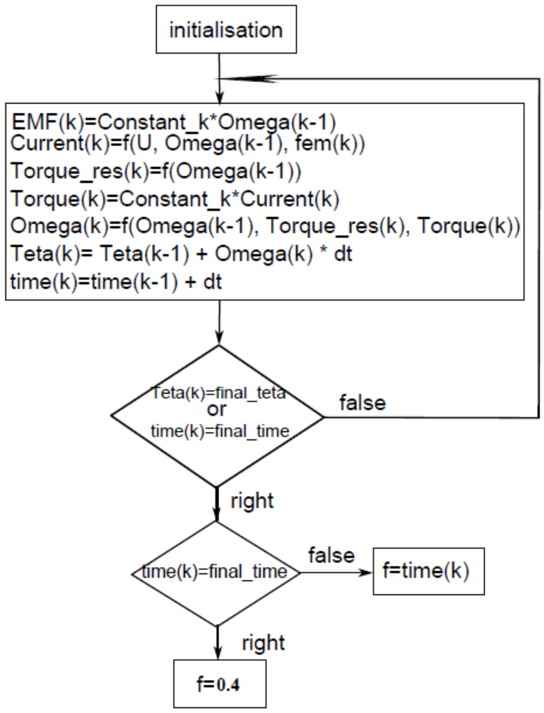


Figure 4. Objective function calculation principle

this comparison is to minimize the calculating time of the objective function when the machine's configuration can't satisfy the predetermined conditions. Consequently, all the machine's configurations which don't satisfy the time criteria will have the objective function's value equals to 0.4. This will create a flat plan as one can see in Fig.5 (as an example, several cases have been computed to show the evolution of the objective function). This figure presents the variation of the objective function with respect to the magnet's depth and the interior radius of the stator.

IV. Introduction to Matlab's optimization algorithms

A. *Sequential Quadratic Programing (SQP)*

This is a gradient-based optimization method. At each iteration, the gradient of the objective function and a linear step are calculated in order to calculate the variables for the next iteration.

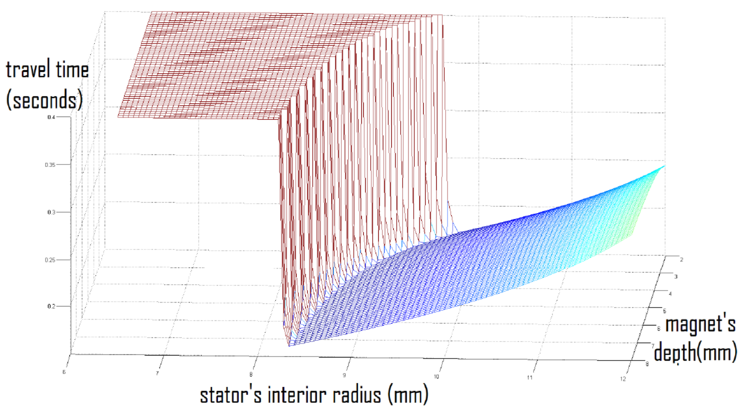


Figure 5. An overview of the objective function

According to [6] the stopping criteria of this algorithm are:

- The distance of the two consecutive points (input vectors of design parameters) is less than X tolerance TolX.
- The change of the objective function between 2 iterations is less than function tolerance TolFun.
- The constraint violation is more than TolCon, which represents the maximum value by which parameter estimates can violate a constraint and still allow successful convergence.
- The number of iterations exceeds MaxIter.

A rapid convergence is the advantage of this algorithm. However, the drawbacks are:

- To use this algorithm, we must specify a starting point; moreover this point must not be far away from the global minimum unless the algorithm will converge to a local minimum. So to use this algorithm, we must have an idea on the zone where the optimal solution could be. To overcome this drawback, the multi-start technique can be used but this also means that the computing time will increase.
- This algorithm can only be used when the objective function is continuous and differentiable, which is not always the case as seen in fig5.

B. *Direct Search (DS)*

Direct search methods belong to a class of optimization methods that do not compute derivatives. Examples of direct search methods are the Hooke and Jeeves' pattern search [7], the Nelder-Mead Simplex method [8], the Dennis and Torczon's parallel direct search algorithm PDS [9] and the Box method [10]. In Matlab, the pattern search method is used.

From an initial point Mi, the algorithm will create a mesh with the central point is Mi then the value of the objective function is evaluated at each node of the mesh. The node Mmin corresponding to the lowest function value is determined. If Mmin and Mi are not the same point, Mmin becomes the initial point for the next iteration. Moreover in the next iternation an expansion step to form a new mesh is carried out in which the size is expanded by some multiple, usually 2. If Mmin and Mi are the same point, then a contraction step to for a new mesh is carried out in which the size is reduced by some multiple, usually 1/2. The algorithm stops when one of the stopping criteria is encountered. According to [6], Direct Search stops when one of these conditions is satisfied:

- The mesh size is less than Mesh tolerance.
- The number of iterations reaches the value of Max iteration.
- The total number of objective function evaluations reaches the value of Max function evaluations.
- The distance between the points found at two consecutive iterations is less than X tolerance.
- The change in the objective function from one successful poll to the next successful poll is less than Function tolerance.

Table II
DIFFERENCE BETWEEN GENETIC ALGORITHM WITH STANDARD ALGORITHMS

Standard Algorithms	*Genetic Algorithm*
Only one point is selected at each iteration. The sequence will approach one optimal solution.	A population is generated at each iteration. The population will approach one optimal solution.
The next point in the sequence is selected by a determinist calculation.	The selection of the next population is partly random.

Because the Direct Search only uses function comparisons, no derivative is needed, this is an advantage compared to the SQP method when encounting problems which are difficult to compute their derivatives. However, as SQP methods there is still a drawback of this method: the initial point must be specified. This point must not be far from the global optimum unless the algorithm will converge to a local optimum.

C. *Genetic Algorithm (GA)*

This third optimization algorithm imitates the natural biological evolution. From an initial population of "individuals", through selection, crossover and mutation, the algorithm will converge to the global optimum. At each iteration, the following procedure is conducted:

1) Choose initial population

2) Evaluate the fitness of each individual in the population

3) Repeat until termination:

- Select best-ranking individuals to reproduce
- Breed new generation through crossover and/or mutation (genetic operations) and give birth to offspring
- Evaluate the individual fitnesses of the offspring
- Replace worst ranked part of population with offspring

The difference between this algorithm and SQP and Direct Search (column standard algorithms) is presented in table II.

According to [6], this algorithm will stop when:

- The number of generations reaches the value of Max Generations.
- The running time in seconds equal to Time limit.
- The value of the fitness function for the best point in the current population is less than or equal to Fitness limit.
- The weighted average change in the fitness function value over Stall generations is less than Function tolerance.
- The algorithm stops if there is no improvement in the objective function during an interval of time in seconds equal to Stall time limit.
- The algorithm runs until the weighted average change in the fitness function value over Stall generations is less than Function tolerance.

The advantage of this algorithm, from the point of view of a user, is that we don't have to specify the initial population, the algorithm itself will do this job. So it is particularly adapted to problems where there is no *a priori* knowledge about the zone where the optimal solution is. But some drawbacks must be mentioned here:

- It is difficult to determine the stopping conditions.
- It is time-consuming because at each iteration, a population of individuals is evaluated.

To conclude, using SQP, Direct Search or Genetic Algorithm depends on the objective function's nature and on the *a priori* knowledge that we have about the optimal solution.

V. Results

We have applied the three algorithms to our objective function with the idea to test which one is the best fitted to this problem.

Since the different variables vary in different ranges which are not homogenous, they are normalized. The normalization consists of transforming the variation of these variables into the interval [0; 1]. The transformation is just a linear one in which the lower bound corresponds to 0 and the upper bound corresponds to 1. This helps the variation of all variables be homogenous.

The following constraints are posed to make sure that the final result is adequate:

- The drive shaft radius must be smaller than the interior radius of the stator minus the depth of the magnet and the air gap.
- The slot area must be positive
- The magnet's depth must be inferior to a ratio of the interior radius of the stator. This ratio is determined to insure that the centrifugal and tangential forces do not pull the magnets out of the rotor.

Once the objective function and the constraint function (which contains inequality expressions and is the same for all the three algorithms that we proceed subsequently) have been made, the optimization procedure can be carried out. The three optimization variables can be presented by a vector $X = [X_1 \quad X_2 \quad X_3]$ where X_1 is the normalized stator's interior radius, X_2 is the normalized magnet's depth and X_3 is the normalized number of conductors per slot. Each normalized variable X_i varies in the interval [0,1]. The other geometric or electric variables are either fixed or dependent on these three optimization variables.

1) Firstly, we will use the SQP method with the Matlab function fmincon. As explained in the section IV-A, this method can only be used when the objective function is continuous and differentiable. this method can not be applied to our problem because of the particularities of our objective function (figure 5): a big jump at the border of the valid zone and a flat invalid zone. Several tests with SQP confirmed this.

2) The search for an optimal configuration of our motor goes on with the utilization of "Direct Search" method. Our starting point is X_0=[0.8182 0.75 0.75] which corresponds to $R_{int-stat}$ = 11mm, l_m = 6.5mm and N = 11, and the stopping criterion is that tolerence over the change of X is 10e-6. We obtained the final value X_f =[0.9312 0.3335 0.2540]. We can see the evolution of the objective function throughout the optimization procedure with the figure 6. A comparison between the starting configuration (X_0) and the final configuration (X_f) is presented in the figure 7. The Direct Search algorithm is robust : the starting configuration (figure 7(a)) chosen out of constraint on permanent magnet's depth allows to reach a final solution which repects all the constraints (figure 7(b)).

3) Finally, we try out the Genetic Algorithm. As presented above, the algorithm does not require a starting point. The stopping criteria are the following:

- The population size is 30.
- Number of generation: 30
- Tolerence on the objective function : 1e-3.

With these criteria, we obtained the vector X_f= [0.5830 0.2818 0.5222] (see figure 8(a)); the value of the objective function is 0.1775.

Cascading method using SQP and GA has been presented in [11]. Here the result from the cascading method of Direct Search and Genetic Algorithm, which means using Direct Search, with the solution given by Genetic Algorithm as the starting point, will be presented. The final result is illustrated in the figure 8(b), the obtained vector is X_f= [0.583 0.2804 0.5226] and the value of the objective function is Fval=0.17745, which is better than the result given by using GA only (0.17745 compares to 0.1775). There is only a small difference

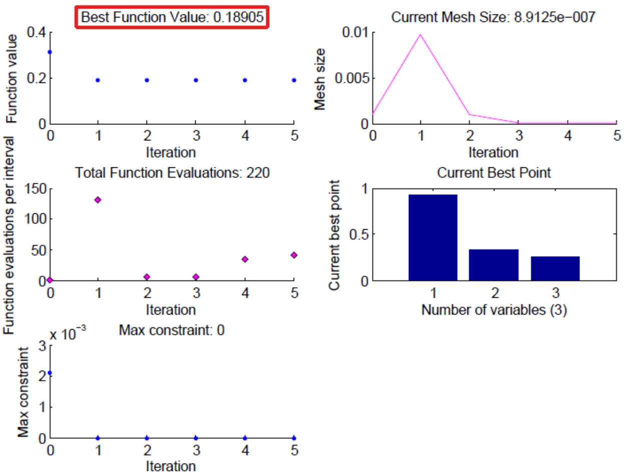
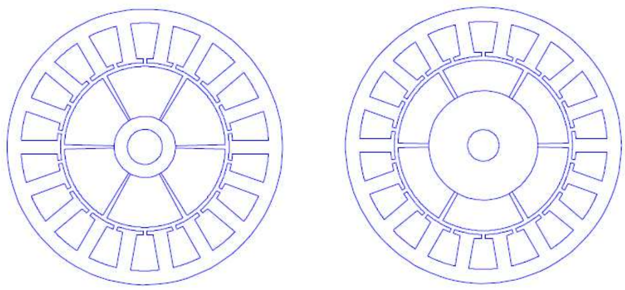


Figure 6. Objective function's evolution with Direch Search method

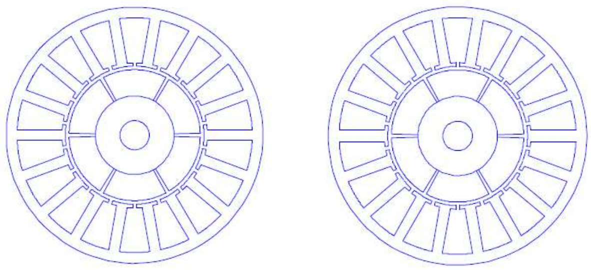


(a) The starting configuration (out of constraints)

(b) Final configuration obtained with Direct Search method

Figure 7. Comparison between the initial and final result which is obtained by Direct Search method

between the two configurations shown in figure 8.



(a) Optimal motor's configuration obtained by Genetic Algorithm

(b) Optimal solution obtained by GA and DS in cascade

Figure 8. Optimal motor's configurations obtained by GA and GA-DS

VI. CONCLUSION

From the results, we can see that in order to satisfy the travel time constraint, the motor can have either a high torque (large interior radius of the stator as shown in the case of Direct Search method) or low rotor inertia (smaller interior radius of the stator as shown in the case of Genetic Algorithm). There is not necessarily one unique good result when optimizing travel time: two optimal solutions can be found, one with a high torque and the other with low rotor inertia. In order to choose the definitive solution, other criteria must be taken into consideration such as losses, costs or heating (out of the scope of this article).

Concerning optimization algorithms, SQP algorithm is not suitable with our particular problem. However it is possible to use either Direct Search or Genetic Algorithm to obtain the optimal solution.

To be able to reach the global optimal solution with Direct Search method, one has to have an a priori knowledge on the zone where the optimal solution could be. This has been shown very clearly in our case: the result obtained by Direct Search has the value of the objective function of 0.18905 (fig 6), whilst the objective function's value for the case of Genetic Algorithm is 0.1775), which means that the starting point for Direct Search does not lie near the optimal solution. This starting point leads to a local optimum.

Another aspect that we have to mention here is that the calculating time for Direct Search is much shorter than the calculating time for Genetic Algorithm (350 seconds compare to 13000 secondes). To overcome this limit, we can use the Genetic Algorithm first, with relatively less strict stopping criteria in order to reduce the calculating time, to obtain the starting point for Direct Search. This cascading utilization of the two algorithms will help to have fully the advantages and avoid the drawbacks of the two algorithms.

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