Prediction of audible magnetic noise radiated by
adjustable speed drive induction machines

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Abstract—This paper presents a fully analytical model of the electromagnetic and vibro-acoustic behavior of variable-speed squirrel-cage induction machines. This model is integrated in a fast simulation tool that can be used to design motors with low magnetic noise level on their whole speed range.

Some simulation results at variable speed are favorably compared with experiments at both vibratory (spectrograms) and acoustic levels (sonagrams) on a 700 W laboratory motor and on an industrial 250 kW traction motor.

These extensive tests show that the model correctly predicts the main exciting forces, motor natural frequencies and resonances during adjustable-speed drive. The simulation tool, called DIVA, has been successfully applied to design a new low-noise ALSTOM motor.

NOMENCLATURE

- \( B_g \): Air-gap radial flux density
- \( D_{so} \): Stator stack inner diameter
- \( E_s \): Stator core young modulus
- \( f_{mm} \): Magnetomotive force
- \( f_{\max} \): Maximum supply frequency
- \( f_n \): \( n \)-th circumferential mode natural frequency
- \( f_R \): Rotor mechanical frequency \((f_R = f_s(1 - s)/p)\)
- \( f_s \): Fundamental stator supply frequency
- \( h_f \): Height of frame
- \( h_{sy} \): Stator height of yoke
- \( n \): Spatial order of a force harmonic
- \( n_n \): \( n \)-th stator circumferential mode
- \( p \): Number of pole pairs
- \( q_s \): Number of stator phases
- \( R_{si} \): Stator stack inner radius
- \( R_{sy} \): Stator yoke mean radius
- \( s \): Fundamental slip
- \( Z_r \): Number of rotor slots
- \( Z_s \): Number of stator slots
- \( \Lambda \): Air-gap permeance per unit area
- \( \mu_0 \): Air-gap magnetic permeability

I. INTRODUCTION

Acoustic noise is a factor that is no longer neglected during design stage of electrical machines, as much for its impact on safety in industrial environments than for its impact on acoustic comfort in electrical transport systems. Noise can be classified in three main sources [1]: mechanical source (bearings, gearbox, etc), aerodynamic source (fan, ventilation ducts, etc) and magnetic source. Magnetic noise is defined as noise coming from vibrations due to magnetic forces which include Maxwell and magnetostrictive forces. It can dominate

the motor global noise level, especially at low and medium speed where fan noise is less important, and it can contain unpleasantly high tonalities that are penalized by IEC 60034-9 norm. It is therefore important to be able to predict high magnetic noise levels in order to design low noise electrical machines.

Fig. I shows the vibration level recorded on an accelerometer sensor placed on the frame of a laboratory 700 W squirrel-cage motor at two different speeds: the vibration level, which is mainly due to magnetic forces at frequencies greater than 500 Hz, strongly depends on the stator supply frequency \( f_s \) as a 30 dB increase is observed near 2400 Hz from \( f_s = 40 \) Hz to 75 Hz. Indeed the distribution of magnetic forces along the air-gap can be represented by an infinite set of rotating force waves (electromagnetic harmonics that can be explained by [2] and [3] produce vibration harmonics coming from the 2D Fourier transform of Maxwell radial forces). These force waves can excite the stator structure natural frequencies as shown in [4]. As the speed of these traveling force waves is proportional to the stator supply frequency, some resonances can appear according to the motor speed value and its stator circumferential modes natural frequencies [5].

In this paper is presented a fully analytical model (DIVA)
of the vibro-acoustic behavior of induction motors, taking into account the interaction between the exciting magnetic forces and the excited structure, and resonances that occur during accelerations and decelerations. Magnetostriction is not of prime importance for machine vibrations [6], [18], therefore this model includes only Maxwell forces. This model is then validated on a wide power range (from 700 W to 250 kW) at both acoustic and vibratory levels, using the sonogram and spectrogram representations.

II. ELECTROMAGNETIC AND VIBRO-ACOUSTIC MODELING

The global effect of Maxwell forces on the stator can be brought to a Maxwell pressure distribution in the air-gap [6]. This pressure \( P_M \) is purely radial on stator teeth, it can be approximated by [7], [8], [9]

\[
P_M = \frac{B_g^2}{2\mu_0}
\]

where \( B_g \) is the radial air-gap flux density. Assuming that the rotor is not skewed, and neglecting end-effects, this force distribution is independent of the motor axial direction. To determine the exciting forces, one must therefore compute the radial air-gap flux density. The analytical model DVA is composed of an electromagnetic model that computes the air-gap flux density, and a vibro-acoustic model that computes the resulting magnetic forces and their impact on vibration and noise.

A. Electromagnetic model

Radial air-gap flux density \( B_g \) can be decomposed as the product of permeance and magnetomotive force (mmf) following

\[
B_g(t, \alpha_s) = \Lambda(t, \alpha_s) f_{mm}(t, \alpha_s)
\]

where \( \alpha_s \) is the angular position in the stator steady frame, \( \Lambda = \mu_0/g_e \) is the air-gap permeance per unit area, \( g_e \) being the effective air-gap magnetic width, \( \mu_0 \) the air-gap magnetic permeability and \( f_{mm} \) the total magnetomotive force. Applying the Ampere’s law to an appropriate path, one can show [3] that

\[
f_{mm}(t, \alpha_s) = \sum_{q=1}^{q_m} i_q(t) N_q^s(\alpha_s) + \sum_{b=1}^{b_m} i_b(t) N_b^r(t, \alpha_s)
\]

where \( N_q^s \) is the 2-D turns function or winding distribution function associated to the stator \( q \)-th phase with current \( i_q \), and \( N_b^r \) is the turns function associated to the rotor \( b \)-th bar with current \( i_b \). This turn function can be easily found from the winding pattern of the motor [10], [11].

The simulated distribution of stator and rotor mmf, permeance and flux density are displayed in Fig. 2 on a 250 kW ALSTOM motor.

B. Vibro-acoustic model

The magnetic pressure \( B_g^2/(2\mu_0) \) is first decomposed in 2D Fourier series to obtain a set of sinusoidal force waves of complex magnitude \( P_{m\omega} \), spatial order \( m \) and pulsation \( \omega \) (see Fig. 3).

The static displacements under each sinusoidal load are then computed by modeling the stator structure as an equivalent ring. This method is also available for other machine types and can be extended to 3 dimension model (axial modes) if necessary [12]. Static displacement associated to order 0 is [9]

\[
Y_{0\omega} = P_{0\omega} \frac{R_{sy} R_{si}}{E_s h_{sy}}
\]

For orders \( m \geq 2 \), static deflections are expressed as

\[
Y_{m\omega} = P_{m\omega} \frac{12 R_{si} P_{sy}^2}{E_s h_{sy}^3 (m^2 - 1)^2}
\]

If we compare the static displacements of order \( m > 1 \) to the zero-th order one, we get

\[
\frac{Y_{m\omega}}{Y_{0\omega}} \propto \frac{P_{m\omega}}{P_{0\omega}} \frac{1}{(m^2 - 1)^2} \frac{R_{sy}^2}{h_{sy}^2}
\]

The ratio \( R_{sy}/h_{sy} \) therefore sizes the ability of the motor to generate high displacements of order \( m > 1 \). Moreover, the higher is the spatial order, the lower is the displacement. This ratio has been plotted for different orders in Fig. 4.
Fig. 4. Relative static displacement in function of $R_{sy}/h_{sy}$ ratio for several modes. Doubling the relative static displacement increases the sound power level at resonance of $10\log_{10}(2^2) \approx 6$ dB.

Once static displacements are determined, one must compute dynamic displacement due to exciting forces. This can be done as a first approximation using a second order filter:

$$Y_d^m(\omega) = Y_s^m(\omega) \frac{(1 - f^2/f_m^2)^2 + 4\xi_m^2 f^2/f_m^2}{}^{1/2}$$

where $f_m$ is the $m$-th stator circumferential mode natural frequency, and $\xi_m$ is the corresponding modal damping coefficient that can be computed using the experimental law established by [13].

These dynamic displacements generate radial velocity vibration waves of magnitude $\omega Y_d^m(\omega)$. From the knowledge of these waves, the global vibration level can be determined [11], as well the global sound power level through a radiation efficiency model [14].

The stator circumferential modes natural frequencies are also computed by assimilating the stator to an equivalent ring [15]. The breathing mode natural frequency $f_0$ is given by

$$f_0 = \frac{1}{2\pi a} \sqrt{\frac{E_s}{K_f a \Delta m \rho_s}}$$

where $a = (D_{so} + h_f - h_{sy})/2$ is the stator mean radius taking into account the frame width $h_f$, $K_f$ is the stator stacking factor, $\rho_s$ the stator stack mass density, and $\Delta m$ is the mass increase due to winding and teeth:

$$\Delta m = 1 + \frac{W_{st} + W_{sw}}{W_f + W_{sy}}$$

where $W_{st}$, $W_{sw}$, $W_f$ and $W_{sy}$ respectively stand for the mass of stator teeth, winding, frame and yoke.

For modes $m > 1$, $f_m$ is computed as

$$f_m = K_m f_0 \frac{h}{2\sqrt{3\pi}} \frac{m(m^2 - 1)}{\sqrt{m^2 + 1}}$$

where $h = h_{sy} + h_f$, and $K_m$ is a corrective coefficient whose expression can be found in [15], [12], [16].

### III. Validations

#### A. Electromagnetic model

The electromagnetic model has been validated by comparing some finite element method (FEM) simulations with FLUX2D software with the analytical model simulations. As an example, Fig. 5 shows the validation on the air-gap radial flux density distribution in time and space on the 250 kW industrial motor.

#### B. Vibro-acoustic model

1) Natural frequencies: The natural frequencies of the stator of the 250 kW industrial motor (see Fig. 6) have been validated by Experimental Modal Analysis (using an external exciting force, a hammer) and FEM [17] (see Table I). Only the even circumferential modes have been considered because on that industrial motor, stator and rotor slots numbers are both even, generating only even spatial orders in the Maxwell exciting force harmonics for symmetry reasons.

Some validations with FEM, Experimental Modal Analysis and Operational Modal Analysis (using the internal exciting
forces of the motor, i.e. magnetic forces) have been also carried on the small 700 W laboratory motor represented in Fig. 7.

Results are displayed in Table II. The deflection shapes of the stator modes measured during the OMA are presented in Fig. 8. A mesh of accelerator sensors is used to determine deflection shapes (mode order and frequency) in three dimensions. This experiment does not show significative influence of axial behavior. Frequencies obtained by three dimensions OMA are closed (±5 percent) to two dimensions theoretical predictions. In that case, odd circumferential modes have been considered as the laboratory motor has some odd numbers of rotor and stator slots [11].

2) Vibration and sound power level: Some acceleration measurements have been done on laboratory motor at variable speed. In that case, measured acceleration spectra at each speed are compiled in a figure called a spectrogram. In a spectrogram (or a sonagram if sound is recorded), time or motor speed (which are generally proportional to the supply frequency) are reported in ordinates, frequencies are reported in abscissa, and the sound or vibration level is indicated with a color gradient. These spectrograms are run at constant flux in order to keep a constant magnetic excitation during starting phase.

Spectrogram Fig. 9, run from 12 to 60 Hz, presents the comparison between measured and simulated vibration spectrogram on the (low noise when sinus fed) 700 W laboratory motor.
motor. We can see that in the lowest part of the spectrum, a high difference exists between DIVA and tests: it is mainly due to mechanical vibrations that are not modeled in DIVA. At higher frequencies ($\geq 500$ Hz), simulation predicts favorably the main vibrations (surrounded) as compared to experimentation. The parts that are amplified independently of motor speed (vertical lines) stand for the motor natural frequencies. We can see that in DIVA spectrogram, the mode 2 natural frequency around 2400 Hz is more amplified than in tests. However, the experimental response of a given natural frequency on a spectrogram highly depends on the position of the accelerometer (nodes or anti-nodes of the mode): the simulation gives an acceleration spectrum averaged over the stator circumference, whereas experiments give an acceleration spectrum on a particular point. In both figures, we can see that the main magnetic vibration lines are well predicted by DIVA. The origin and the expressions of all these vibration lines (winding, slotting and saturation harmonics) is detailed in [18], [19]. Frequencies and spatial orders of main vibrations are presented in Table III.

<table>
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<tr>
<td>11</td>
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TABLE III
CHARACTERISATION OF MOTOR M1 ($Z_s = 27$, $Z_r = 21$, $p = 2$) PURE SLOTTING VIBRATION LINES.

After reliable prediction of vibrations on low noise laboratory motor, validations must be extended to noise on industrial motor. Fig. 10 shows an experimental and simulated sonagram of the 250 kW industrial motor run from 0 to 200 Hz. Note that although the colorbars are the same, the scales are different as the experimental sonagram has a minimum ambient noise of 50 dB, whereas simulated sonagram starts from 0 dB. The main natural frequencies (vertical line) near 720 Hz, which corresponds to the stator elliptical mode (cf. Table I), and the main magnetic lines (depending on supply frequency) are correctly predicted by DIVA. Major source of noise caused by coincidence of magnetic lines and mechanical resonance (near 50 Hz supply frequencies) is also well predicted (surrounded). The origin and the expressions of all these acoustic lines are detailed in [18], [20].

IV. APPLICATION

On the ground of this simulation tool, a new 250 kW ALSTOM motor has been built by changing the rotor design: the numbers of rotor slots has been chosen to avoid resonances due to slotting harmonics [5] and saturation harmonics [21], and the rotor slot opening width has been fixed to reduce some
magnetic force harmonics magnitude [22]. Fig. 11 shows this rotor prototype.

A 15 dB noise reduction was obtained during starting in off-load case (compared to a simulated 20 dB reduction [18]). The expected tendency is also well respected in on-load case with a 10 dB decrease as shown Fig.12. The simulation tool has therefore been successfully applied to drastically reduce the magnetic noise level in the whole speed range.

V. CONCLUSION

A fully analytical model of the induction machine, describing its electromagnetic and vibro-acoustic behavior, has been described. This model allows predicting the audible magnetic noise level radiated by the motor, and principal resonances occurring during starting or braking. The model has been validated at different stages, with both experiments and FEM, and on a wide power range (from 700 W to 250 kW). This tool can be used for both designing new low-noise motors, and diagnose magnetic noise issues on existing motors. It is especially fast and accurate (a spectrogram going up to 20 kHz is obtained in a few minutes on a 2GHz laptop) without coupling computationally greedy numerical methods or using statistical energy method [23], [24].

This simulation tool has been applied to design a new 250 kW traction motor with a significant noise reduction.

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REFERENCES