Bayesian identification of acoustic impedance in treated ducts

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Abstract: The noise reduction of a liner placed in the nacelle of a turbofan engine is still difficult to predict due to the lack of knowledge of its acoustic impedance that depends on grazing flow profile, mode order, and sound pressure level. An eduction method, based on a Bayesian approach, is presented here to adjust an impedance model of the liner from sound pressures measured in a rectangular treated duct under multimodal propagation and flow. The cost function is regularized with prior information provided by Guess's [J. Sound Vib. **40**, 119–137 (1975)] impedance of a perforated plate. The multi-parameter optimization is achieved with an Evolutionary-Markov-Chain-Monte-Carlo algorithm.

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1. Introduction

The characterization of acoustic treatments placed in aircraft engines remains a challenge for the design of liners able to provide high reduction of radiated noise. This is the reason why experimental eduction procedures able to fully reproduce in a laboratory the real aero-acoustics environment of the liner have been developed.

These inverse methods[1,2] are mainly based on the minimization of a cost function that evaluates the $L2$-norm between simulated and measured data. This function depends on the impedance to identify and leads to a complex optimization problem at each frequency. A three-dimensional finite element model based on the convected Helmholtz's equation with a uniform flow was used with the Ingard-Myers's coupling condition to fit experimental data in previous works.[1] The optimization problem was also solved with the adjoint-based method[3] without the usual cumbersome calculations necessary to find the optimum. Other inverse methods were proposed[4–6] where the cost functions were based either on the sound pressures at the walls,[5] the scattering matrix coefficients,[4] or the power dissipated by the liner,[6] while the direct models rely either on a mode matching method,[4,5,7] or an axisymmetric finite element method.[6] In all these studies the impedance is always adjusted step by step at each frequency.

A well posed inverse problem and a robust optimization tool are then necessary to get correct results from uncertain aero-acoustics conditions. The Bayesian framework, known for its robustness and its ability to regularize inverse problems[9] in an elegant way is, in this paper, developed to determine the impedance of a liner. A parametric model describing the liner impedance by a set of five parameters in the frequency domain has been chosen. A physical meaning of the impedance can hence be added as the classical $\cot(kH)$ term accounting for the honeycomb cavity and a function representing the perforated plate.

In addition, with this formalism, prior information is added on each parameter by the user from its knowledge or from a previous experiment resulting in a cost function automatically regularized.

On one hand, an analytical model based on the convected Helmholtz's equation is achieved. On the other hand, experiments are carried out in a rectangular duct with flow and multimodal propagation conditions. An Evolutionary Monte Carlo Markov Chain (EMCMC) method is finally used to optimize the Bayesian cost function in a robust way, and to estimate the impedance on the whole frequency band with a parametric model.

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2. Problem description

Let us consider a lined duct with a rectangular cross section as depicted in Fig. 1 (dimensions $a = 0.2$ m and $b = 0.1$ m). The locally reacting liner, made of a perforated plate (thickness 0.5 mm, hole diameter 0.45 mm, perforation rate 3.12%), and a 35 mm thick honeycomb layer backed with a rigid plate, is placed on the upper face of the test section (length 0.31 m).

Multimodal propagation conditions up to 2500 Hz are applied with a grazing mean flow. The terminations of the experimental facility are not perfectly anechoic, but the reflection conditions at both ends have been measured and can be taken into account in the propagation model.

3. Mode matching model

In order to use the Bayesian inference to estimate the surface impedance, an analytic model simulating the propagation of complex acoustical waves in a rectangular duct made of two rigid wall parts (I and III) located on each side of the test section (II) is implemented with a mean flow assumption. The measured reflecting conditions at both ends of the duct (see Fig. 1) are also taken into account. The model is based on a modal expansion and a mode-matching method assuming a $e^{+i\omega t}$ time dependency. The acoustical pressure in each duct section, $s = $ I, II, III, is expanded over its modal basis in the form:

$$p^s = \sum_{mn}^{K} A_{mn}^{s+} \phi_{mn}^{s+}(x,y) e^{ik_{mn}^{+} z} + A_{mn}^{s-} \phi_{mn}^{s-}(x,y) e^{ik_{mn}^{-} z}, \quad (1)$$

with $A_{mn}^{s\pm}$ the modal amplitudes of the incident and reflected waves in the section s, and $\phi^s(x,y)_{mn}$ the related pressure (m,n) mode shape. The acoustical velocity writes in the same way:

$$v^s = \sum_{mn}^{K} A_{mn}^{s+} \varphi_{mn}^{s+}(x,y) e^{ik_{mn}^{+} z} + A_{mn}^{s-} \varphi_{mn}^{s-}(x,y) e^{ik_{mn}^{-} z}, \quad (2)$$

with $\varphi_{mn}^{s}(x,y)$ the related velocity (m,n) mode shape in the duct section s. In the rigid part, the transverse wavenumbers $k_m = (m\pi/a)$ and $k_n = (n\pi/b)$ are related to the wavenumber $k = \omega/c$ and to the axial wavenumber $k_{mn}^{\pm}$ convected at a Mach number M by the dispersion relationship

$$k_{mn}^{\pm} = \frac{kM \mp \sqrt{k^2 - (1 - M^2)(k_m^2 + k_n^2)}}{1 - M^2}. \quad (3)$$

In the treated part, the Ingard-Myers condition is used at the liner interface $(y = b)$:

$$k_n \sin(k_n b) - \frac{i\rho_0 c_0 (k + k_{mn} M)^2}{kZ} \cos(k_n b) = 0, \quad (4)$$

with ρ_0 the air density and c_0 the sound speed. The Newton-Rhapson's method is then used to get the wavenumber k_n in the test section with the rigid mode wavenumber $k_n = (n\pi/b)$ as initial value.

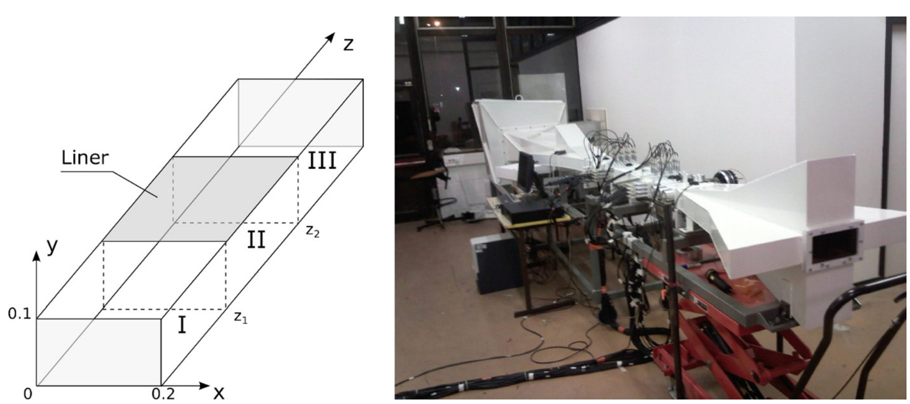

Fig. 1. (Color online) Lined duct submitted to a grazing flow and a multimodal propagation.

The pressures in the rigid and treated parts are now coupled with a mode-matching technique[7] briefly presented here. The pressure and velocity continuity conditions at each cross section interface, i.e., $p^s = p^{s+1}$ and $v^s = v^{s+1}$, lead to a system of equations between the unknown modal amplitudes in each section. In practice, the matching conditions are formulated in a variational form and integrated in the cross sectional area of the transition planes. Also, according to Nennig *et al.*[7] the most efficient choice for the test functions are the rigid wall mode shapes. Finally, the remaining unknowns of the problem are the reflected and transmitted modal amplitudes while the incident modal amplitudes $A^{\mathrm{I}+}$ and $A^{\mathrm{III}-}$ are given as input. A set of incident propagating modes are indeed applied in the first section, and the measured reflection coefficients of the anechoic terminations are used at both ends. A scattering system between the unknown is hence obtained. An iterative method is performed to solve this system with a given convergence criterion. This analytical model, which better suits an optimization problem, has been first validated by comparing the results with a numerical model for non-reflecting end conditions.

4. Bayesian identification method

In the identification process described here, the impedance Z is parametrized by the function

$$Z(f) = (af + b) + \mathrm{i}(cf + d) - \mathrm{i}\cot(kH), \tag{5}$$

with a set of five parameters $\theta = (a, b, c, d, H)$ which are the unknowns of the problem. The resistance and reactance of the perforated plate are modeled here as a linear function of the frequency f while $\cot(kH)$ is related to the well known honeycomb cavity impedance. This model catches the main physical phenomenon (see Fig. 2) and does not decrease the method accuracy. The conditional probability density function (pdf) of the parameters θ given the measured pressures $\tilde{P}_{ik}$ is noted $p(\theta|\tilde{P}_{ik})$, where i is the microphone index and k is the frequency index, and is also called the posterior pdf. Using Bayes's Theorem, the posterior pdf is expressed as:

$$p(\theta|\tilde{P}_{ik}) = \frac{p(\tilde{P}_{ik}|\theta)p(\theta)}{p(\tilde{P}_{ik})}. \tag{6}$$

The pdf $p(\tilde{P}_{ik})$ is called the evidence. It does not depend on the parameters θ and is therefore constant in Eq. (6). The prior pdf $p(\theta)$ is the user's knowledge on the parameters before the experiment. It is very important in the Bayesian approach since it leads to the stability and the uniqueness of the solution. At the first approximation, all the parameters are assumed independent *a priori* and constant for each frequency and microphone, i.e., $p(\theta) = \prod_j^{n_j} p(\theta_j)^{n_i n_k}$, but not necessarily *a posteriori*. A Gaussian pdf is also used in a first step to characterize each $p(\theta_j)$. Finally the likelihood function $p(\tilde{P}_{ik}|\theta)$ is the probability to have the measurement knowing the parameters. It is evaluated from the direct problem with the analytic model presented previously. The measured pressures are hence written as the sum of a predicted pressure $P_{ik}(\theta)$ with a set of parameters θ and a random noise N_{ik}:

$$\tilde{P}_{ik} = P_{ik}(\theta) + N_{ik}. \tag{7}$$

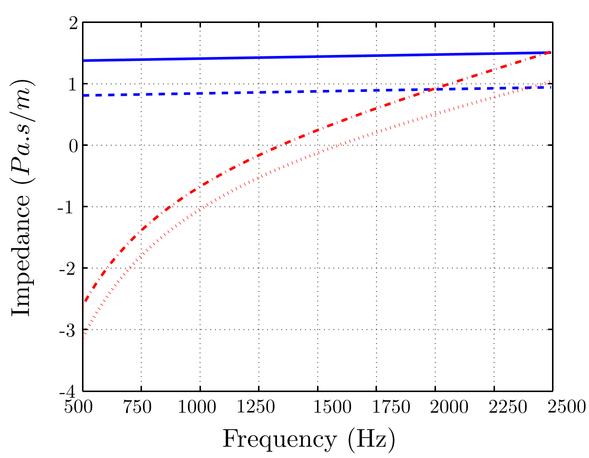

Fig. 2. (Color online) Comparison of the prior impedance value with the mean posterior impedance. (——) Prior resistance, (- - - - -) posterior resistance, (- - -) prior reactance, (·····) posterior reactance.

The likelihood pdf, related to the noise N_{ik}, is then expressed with a complex Gaussian pdf by invoking the Central Limit Theorem.

With all these assumptions, the posterior pdf reduces to:

$$p(\theta|\tilde{P}_{ik}) \propto \left(\prod_j^{n_j} \frac{1}{\sqrt{2\pi}\,\sigma_{\theta_j}} \exp\left(-\frac{|\theta_j - \bar{\theta}_j|^2}{\sigma_{\theta_j}^2}\right)\right)^{n_i n_k} \prod_k^{n_k} \prod_i^{n_i} \frac{1}{\pi\sigma_{ik}^2} \exp\left(-\frac{|P_{ik} - \tilde{P}_{ik}|^2}{\sigma_{ik}^2}\right),$$

with $\bar{\theta}_j$ the prior value of the parameter j, σ_{θ_j} is the standard deviation, and σ_{ik} the noise standard deviation. The optimal parameters are obtained by maximizing the posterior pdf or in a more convenient way by minimizing its negative logarithm. This leads to the following cost function:

$$\mathcal{J}(\theta) = \sum_j^{n_j} \frac{|\theta_j - \bar{\theta}_j|^2}{\sigma_{\theta_j}^2} + \frac{1}{n_i n_k} \sum_k^{n_k} \sum_i^{n_i} \frac{|P_{ik} - \tilde{P}_{ik}|^2}{\sigma_{ik}^2}. \tag{8}$$

The prior parameter values ($\bar{\theta}_j$ such as $a = 6.52 \times 10^{-5}$; $b = 1.35$; $c = 5.55 \times 10^{-4}$; $d = 0.11$; $H = 35 \times 10^{-3}$ m) are obtained by fitting the function (5) on Guess's model.[8] Other models or experiments could be used instead but would not change the results as long as the prior impedance is not too far from the real impedance, and as long as there is enough posterior information (enough measured data).

An EMCMC technique[9] based on *parallel tempering* is then used to explore the space of the parameters pdf. This method combines a Genetic Algorithm, a Monte Carlo method, and several Markov Chains to explore the search domain in a smart way. The idea behind EMCMC methods is to generate a range of Markov Chains with a Metropolis-Hastings algorithm that converges to the posterior pdf. At each generation in the Markov Chains, a Genetic Algorithm is used to exchange information between the different chains by crossovers and permutations. This avoids being trapped in a local minimum of the cost function. Contrary to classical optimization methods, the EMCMC method thus gives a pdf estimation for each parameter.

5. Results

The method is now applied to the determination of the liner impedance as described in Fig. 1. The acoustic pressures produced by an acoustic driver located upstream of the lined section are measured with 3×12 microphones on each side of the test section in order to get the modal expansions of the incident and reflected waves and the experimental input data of the eduction technics. The experiment is conducted with a positive 44 m/s mean flow velocity in the frequency range [500–2500 Hz] where the liner is known to be efficient and where at least 2 modes can be cut-on (the first non-plane wave mode is cut-on at 860 Hz).

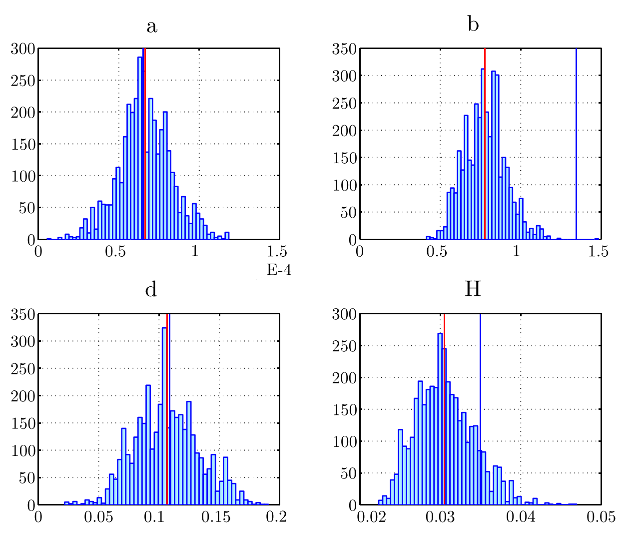

Fig. 3. (Color online) Posterior pdfs of the impedance parameters in Eq. (5) obtained with the EMCMC algorithm. Reference values (blue vertical lines) and mean posterior values (red vertical lines) are displayed for each parameter.

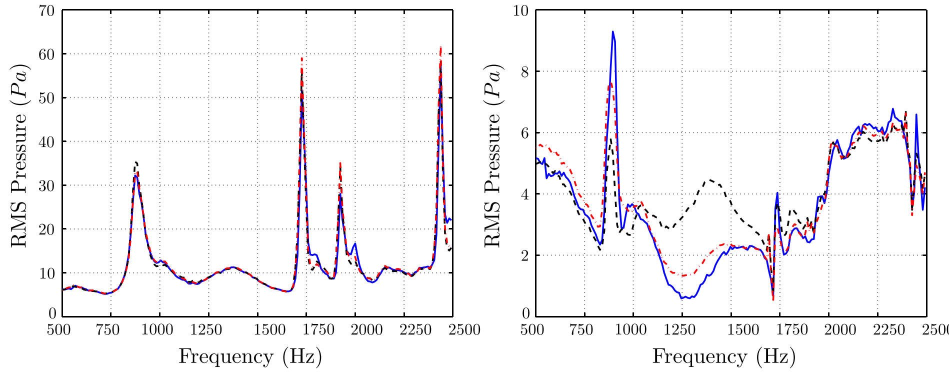

Fig. 4. (Color online) Comparison of the averaged rms pressure over the 36 microphones in sections I (left) and III (right). (——) Measured pressure, (- - - - -) Calculated pressure with the prior parameters (- - -) Calculated pressure with the mean posterior parameters.

The standard deviations σ_{θ_j} and σ_{ik} are set to 25% and 30%, respectively. These choices are made in order to balance the impact of the likelihood function with the prior pdf on the posterior pdf.

To explore the parameter space, the EMCMC is applied with 10 individual chains on 5000 generations. In order to guarantee the convergence of the chains, the first 1250 generations are eliminated. The initial parameter values are defined far from the prior values in order to explore the whole space of the pdfs. The posterior pdfs are presented in Fig. 3 and show that the number of generations is sufficient. The posterior parameters are estimated here with the mean values of each posterior pdf instead of the *maximum a posteriori*. The estimated parameters are then compared with the initial parameters. The difference between prior and posterior values are +2% for the parameter a, −42% for b, −20% for c, −2% for d, and −13% for H. Note that the physical parameter H should not be viewed as the real cavity depth but as an equivalent length that includes all the modeling errors and experimental uncertainties.

The posterior impedance is also compared to the prior impedance in Fig. 2 showing that the prior impedance evaluated by Guess's model is overestimated on the whole frequency range. Finally, Fig. 4 presents a comparison between the measured and calculated pressures (with the initial and final estimated parameters). This comparison is made on the averaged root-mean-square (rms) pressure over the 36 microphones upstream and the 36 microphones downstream. Note that the high pressure levels concerned by non-linearities are localized at the mode cutting frequencies and have been removed from the calculations. Finally the estimated parameters give a very good estimation of the pressure in the duct, much better than the initial estimation with the impedance given by Guess's model.

6. Conclusion

The method presented in this paper is part of the so-called *impedance eduction methods* very popular in aeronautics to characterize the acoustic liners properties. In general the eduction process is based on the minimization of a cost function representing the distance between a model and experimental data. Here, contrary to classical methods, the impedance is not adjusted step by step at each frequency but at once on the whole frequency range with a parametric model. This enables to add physical constraints to the liner impedance. The cost function based on a mode matching model is also given in a Bayesian framework. Prior information is hence added to regularize the inverse problem under uncertain aero-acoustics conditions. An EMCMC algorithm is then used to explore the solutions space. In this paper the method is applied successfully to measured pressures with flow in multimodal propagation conditions and a very good reconstruction of the sound pressures is achieved in the treated duct.

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