# Mode matching method for cylindrical dissipative silencers with poroelastic material

**B. Nennig**[1], **E. Perrey-Debain**[1,*], **M. Ben Tahar**[1]
[1] Laboratoire Roberval, Université de Technologie de Compiègne
60205 Compiègne BP 20529 France
*Email: emmanuel.perrey-debain@utc.fr

## Abstract

We develop a new mode matching method for predicting the transmisssion loss (TL) of a cylindrical shaped dissipative silencer. The sound propagation through the absorbing poroelastic material is described via Biot's theory. The semi-analytical model provides results which compare favorably with FEM calculations.

## Introduction and problem statement

Typical dissipative silencers encountered in the automotive industry consist of an expansion chamber filled with a sound absorbing material. Elastic porous materials such as foams can convey two longitudinal and one transverse wave. In a recent communication, the influence of the elastic phase in a poroelasic material was numerically investigated in [3]. In particular, it was found that shear wave resonance effects can have a noticeable effect on the TL. For silencers of cylindrical shape, mode-matching techniques are efficient tool for TL predictions. These semi-analytical methods, though very efficient compared to standard 3D finite element models, have been developed and tested in the scalar case where only pressure wave types are present. In this work, we present a new mode matching method (MMM) which is valid for any type of poroelastic material.

The silencer considered here consists of a cylindrical chamber duct containing mean flow (with the usual constant parameters $U_0, \rho_0$ and $c_0$) surrounded by a poroelastic material (region II in Fig. 1). The inlet and outlet pipes (regions I and III) are identical with rigid walls.

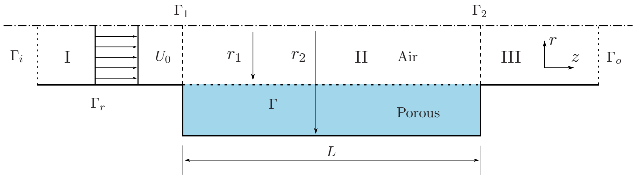

Figure 1: Geometry of the silencer.

## 1 Mode calculation

In this section, the silencer chamber is assumed first to be infinite in length. The eigenvalue analysis is performed by assuming that all perturbative quantities have the $\mathrm{e}^{\mathrm{i}(\beta z-\omega t)}$ dependence. Within these assumptions, the displacement perturbation $\mathbf{w}$ satisfies the convected wave equation in the airflow domain

$$\nabla\nabla\cdot\mathbf{w}+\Omega^2\mathbf{w}=0 \tag{1}$$

with $\Omega=M\beta-k_0$, $k_0=\omega/c_0$ and $M=U_0/c_0$. Since the flow is uniform, we can look for purely acoustic mode by putting $\mathbf{w}=\nabla\varphi_0$ and the acoustic pressure is simply obtained from $p=\rho_0 c_0^2\Omega^2\varphi_0$. In the axisymmetric case, we find that $\varphi_0(r)=A_0J_0(\alpha_0 r)$ where the transverse wavenumber satisfies the dispersion relation $\alpha_0^2+\beta^2=\Omega^2$. The fluid and solid phase displacements (resp. $\mathbf{U}$ and $\mathbf{u}$) in the porous materials are governed by the Biot's model. When the medium is homogeneous, both displacements admit the Helmholtz decomposition $\mathbf{u}=\nabla(\varphi_1+\varphi_2)+\nabla\times\boldsymbol{\Psi}$ and $\mathbf{U}=\nabla(\mu_1\varphi_1+\mu_2\varphi_2)+\mu_3\nabla\times\boldsymbol{\Psi}$ where potentials $\varphi_1$, $\varphi_2$ and $\boldsymbol{\Psi}$ are solutions of the Helmholtz equation with wavenumbers $k_1$, $k_2$ and $k_3$, (see [1]). So we put

$$\begin{array}{rcll}\varphi_e(r) &=& A_eJ_0(\alpha_e r)+B_eY_0(\alpha_e r), & e=1,2, \quad (2)\\ \varphi_3(r) &=& A_3J_1(\alpha_3 r)+B_3Y_1(\alpha_3 r), & \quad (3)\end{array}$$

where $\alpha_e^2+\beta^2=k_e^2\,(e=1,2,3)$. Note that, for the sake of simplicity, the shear wave potential $\varphi_3$ simply stands for the azimuthal component of $\boldsymbol{\Psi}$. The modal vector $\mathbf{V}=[A_2,\,B_2,\,A_3,\,B_3,\,A_1,\,B_1,\,A_0]^T$ must be found so that the following conditions at the fluid-porous interface ($r=r_1$) are satisfied : $\boldsymbol{\sigma}^t\mathbf{n}=-p\,\mathbf{n}$, $p_p=p$ and $\phi(\mathbf{U}-\mathbf{u})\cdot\mathbf{n}+\mathbf{u}\cdot\mathbf{n}=\mathbf{w}\cdot\mathbf{n}$. Here $\phi$ is the porosity and $\mathbf{n}$ denotes the normal unit vector at the interface. The pore pressure $p_p$ and the total stress tensor $\boldsymbol{\sigma}^t$ can be expressed in terms of the potentials as shown in [1]. Similarly, at the wall ($r=r_2$), the foam is clamped, i.e. $\mathbf{u}=0$ and $\mathbf{U}\cdot\mathbf{n}=0$. All these conditions leads to the nonlinear eigenvalue problem

$$\mathsf{M}(\beta)\mathbf{V}=\mathbf{0}, \tag{4}$$

where $\mathsf{M}(\beta)$ is a 7×7 matrix. We can now exploit the fact that $\det \mathsf{M}(\beta)$ is a meromorphic function in the complex $\beta$-plane and solve $\det \mathsf{M}(\beta) = 0$ using the *argument principle*. All simple zeros $\beta_k^{\pm}$ $(k = 1, \ldots, K)$ are classified with the convention that superscript ± stands for the sign of the imaginary part (+ refers to rightgoing modes and - to leftgoing modes).

## 2 Mode Matching and results

In regions $i$=I,II,III, each quantity is expanded via its truncated modal decomposition. For instance, the acoustic pressure is expressed as

$$p^i = \sum_{k=1}^{K} \left( \mathrm{e}^{\mathrm{i}\beta_k^{i,+} z} \Phi_{p,k}^{i,+}(r) A_k^{i,+} + \mathrm{e}^{\mathrm{i}\beta_k^{i,-} z} \Phi_{p,k}^{i,-}(r) A_k^{i,-} \right)$$

Here, $A_k^{i,\pm}$ are the modal amplitudes and $\Phi_{p,k}^{i,\pm}(r)$ are the corresponding radial eigenfunctions, subscript $p$ indicates the physical quantity (here the pressure) it is associated with. A similar decomposition holds for the displacements $\mathbf{U}, \mathbf{u}$ and $\mathbf{w}$. The acoustic axial displacement as well as the pressure are to be matched a $z = 0$ and $z = L$, i.e. $p^{\mathrm{I}} = p^{\mathrm{II}}$ and $\mathbf{w}^{\mathrm{I}} \cdot \mathbf{n} = \mathbf{w}^{\mathrm{II}} \cdot \mathbf{n}$ $(0 \leq r \leq r_1)$. On the rigid wall $(r_1 \leq r \leq r_2)$, the foam is clamped, i.e. $\mathbf{U}^{\mathrm{II}} \cdot \mathbf{n} = 0$ and $\mathbf{u}^{\mathrm{II}} = \mathbf{0}$ (note the normal unit vector $\mathbf{n}$ is now axial). These conditions are imposed in a weighted sense, for instance, at the inlet plane $z = 0$, we put

$$\int_0^{r_1} \Psi_p \, (p^{\mathrm{I}} - p^{\mathrm{II}}) \, r \, \mathrm{d}r = 0, \tag{5a}$$

$$\int_0^{r_1} \Psi_w \, (\mathbf{w}^{\mathrm{I}} - \mathbf{w}^{\mathrm{II}}) \cdot \mathbf{n} \, r \, \mathrm{d}r = 0, \tag{5b}$$

$$\int_{r_1}^{r_2} \mathbf{\Psi_u} \cdot \mathbf{u}^{\mathrm{II}} \, r \, \mathrm{d}r + \int_{r_1}^{r_2} \Psi_{U_z} \mathbf{U}^{\mathrm{II}} \cdot \mathbf{n} \, r \, \mathrm{d}r = 0, \tag{5c}$$

and similarly at the exit plane $z = L$. Here, weighting functions $\Psi_p, \Psi_w, \Psi_{U_z}$ and $\mathbf{\Psi_u}$ are chosen among the eigenfunctions associated with each physical variable. After modal substitution, we obtain the two scattering systems

$$\mathbf{X}_1 \begin{pmatrix} \mathbf{A}^{\mathrm{I},-} \\ \mathbf{A}^{\mathrm{II},+} \end{pmatrix} = \mathbf{Y}_1 \begin{pmatrix} \mathbf{A}^{\mathrm{I},+} \\ \mathbf{A}^{\mathrm{II},-} \end{pmatrix}, \tag{6a}$$

$$\mathbf{X}_2 \, \mathbf{E}_X \begin{pmatrix} \mathbf{A}^{\mathrm{III},+} \\ \mathbf{A}^{\mathrm{II},-} \end{pmatrix} = \mathbf{Y}_2 \, \mathbf{E}_Y \begin{pmatrix} \mathbf{A}^{\mathrm{III},-} \\ \mathbf{A}^{\mathrm{II},+} \end{pmatrix}, \tag{6b}$$

where $\mathbf{E}_X$ and $\mathbf{E}_Y$ are diagonal matrices containing the propagation terms $\mathrm{e}^{\mathrm{i}\beta_k^{i,\pm} L}$. The above coupled system is then solved using an iterative scheme as in [2], and $\mathbf{E}_X$ is inverted analytically in order to reduce possible round-off errors associated with the presence of strongly evanescent modes. As underlined in [2], the choice of the weighting function is essential. In order to improve the conditioning of $\mathbf{X}_j$ $(j = 1, 2)$ it is preferable to use different weighting functions for each equation of (5) to maximise the diagonal terms in (6). Thus rigid wall modes (of region I and III) are used in Eq. (5a) and lossy modes of region II in Eq. (5b-5c). In Fig. 2 are shown the computed TL in the case of an incident plane wave. Results clearly show very good agreements with the FE method. The XFM foam was chosen to illustrate the presence of shear waves (identifiable at the peaks). In this example, a typical fluid equivalent model can not produce reliable results [3].

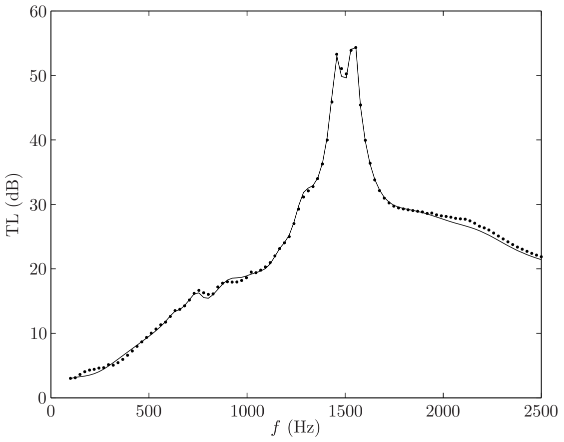

Figure 2: TL for silencer A with a $M = 0.2$ flow and XFM foam, —— 78,000 dofs 3D FE calculation [3], · MMM with 15 modes

### References

[1] J.-F. Allard. *Propagation of Sound in Porous Media: Modeling Sound Absorbing Materials.* Chapman & Hall, 1993.

[2] G. Gabard and R. J. Astley. A computational mode-matching approach for sound propagation in three-dimensional ducts with flow. *J. Sound Vib.*, 315:1103–1124, 2008.

[3] B. Nennig, J.-D. Chazot, E. Perrey-Debain, and M. Ben Tahar. Influence of solid phase elasticity in poroelastic liners submitted to grazing flows. In *ACOUSTICS'08*, Paris, France, July 2008.