TE mode propagation in curved multimode waveguides

E. Perrey-Debain†,‡, I.D. Abrahams††

†Laboratoire Roberval, Université de Technologie de Compiègne
60205 Compiègne BP 20529 France
‡‡School of Mathematics, University of Manchester,
Oxford Road, Manchester, M13 9PL, UK
*Email: emmanuel.perrey-debain@utc.fr

Abstract

Based on an improved coupled mode theory involving the notion of ‘bend’ modes, we obtain a new form for the propagation kernel for the light beam as it travels along a weakly curved waveguide. This allows an efficient numerical evaluation of the optical field for waveguides supporting a very large number of modes.

Introduction

Large multimode fibres are finding increasing application in many areas of applied science. In practice, macrobending occurs in a large deflection of the fibre axis such as that associated with spooling or the presence of loops. These deviations influence the signal propagation as a result of mode coupling phenomena. Because standard coupled mode theory becomes numerically intractable when the number of propagative modes is too large (plastic optical fibres, for instance, can support several hundred thousand modes), there is a need for devising new strategies. In this work, we propose an improved coupled mode theory whereby the coupling mechanism is described via ‘bend’ modes. Though restricted to planar waveguides, the theory presented here is a starting point for the analysis of curved round fibres.

1 Problem statement

We aim to study the propagation of a monochromatic TE (transverse electric) field $E = \hat{E} e^{-i\omega t}$ in a weakly guiding two-dimensional dielectric waveguide whose graded-index profile $n$ in the core of width $2a$ has the parabolic form

$$n^2(u) = n_0^2 \left(1 - 2\Delta(u/a)^2\right), \quad |u| \leq a, \quad (1)$$

where $\Delta = (n_0^2 - n_c^2)/2n_0^2$ denotes the usual profile height parameter, $n_0$ is the usual profile height parameter, $n_c$ the index of the cladding. Equation (1) is written in the local coordinate system $r(s, u) = r_0(s) + un$ where $r_0(s)$ is a point of a smooth curve $C$ following the centre of the waveguide along its bent path as indicated in Figure 1 and $s$ is the arc length. The local basis vectors are defined by $t = \partial_s r_0$ and $n = \kappa^{-1} \partial^2_{ss} r_0$ where $\kappa$ is the curvature. We assume that the angle $\phi$ between a tangent to the centreline and the horizontal line is a smooth function of a slow variable $\sigma = \varepsilon s/a$, where $\varepsilon$ is a small dimensionless parameter may be thought of as a ratio of the half-width $a$ to a typical radius of curvature.

By expressing the Laplacian operator in the new coordinate system, we find that, to first order in $\varepsilon$, the envelope of the electric field $\Psi = \hat{E} \exp(-i\kappa n_0 s)$ ($k$ is the vacuum wavenumber) must satisfy

$$\delta \partial^2_{zz} \Psi + i \partial_z \Psi = \mathcal{H} \Psi + \gamma x \Psi + \mathcal{O}(\varepsilon). \quad (2)$$

Equation (2) has been normalized under the appropriate scaling $x = u \sqrt{V}/a$ and $z = s \sqrt{2\Delta}/a$, where $V$ stands for the usual waveguide parameter $V = kn_0 a \sqrt{2\Delta}$. In (2), $\delta = \Delta/V$ characterizes the strength of the second $z$-derivative paraxial term. The transverse operator $\mathcal{H}$ is the $z$-independent Hamiltonian

$$\mathcal{H} = -\frac{1}{2} \left(\partial^2_{xx} - v\right) \quad (3)$$
corresponding to the straight waveguide. The potential \( v \) stands for the quadratic well of finite depth \( v(x) = x^2 \) where \( |x| \leq \sqrt{V} \) and \( v(x) = V \) otherwise. Thus, the interface core-cladding is now located at \( |x| = \sqrt{V} \). Function \( \gamma \) can be interpreted as the normalized curvature within this new coordinate system. It depends on the slow variable \( \sigma \) and is defined as \( \gamma(\sigma) = \eta \partial_\sigma \varphi \) with \( \eta = \epsilon/(2\sqrt{2}\delta) \) and \( \epsilon = \varepsilon/(\sqrt{2}\Delta) \) (note \( \sigma = \varepsilon z \)). As we are dealing with very large mode area optical waveguides (i.e. \( V \gg 1 \)) operating in the weak guidance limit (\( \Delta \ll 1 \)), the problem becomes

\[
i \partial_\nu \Psi = \Theta \Psi + \gamma x \Psi. \tag{4}
\]

2 Standard coupled mode theory

Equation (4) is nothing else but the Schrödinger equation which describes the motion of a harmonic oscillator in the presence of a varying external force. In the present study we consider an optical wave field which is well confined within the core region and one can assume that the parabolic profile extends to infinity. If this is done then the solution of the initial value problem (4) is obtained via the Feynman’s propagator [1]

\[
\Psi(x,z) = \int_{\mathbb{R}} G(x,z;x',0)\Psi(x',0)dx'. \tag{5}
\]

Here the Green function is given by \( G = G_0 e^{iS} \) where \( G_0 \) is the fundamental solution of the straight waveguide and curvature effects are described by the phase function \( S \). Despite its elegance, the evaluation of the integral (5) is not trivial in this form. Another classical method for solving (4) is to express the field distribution in the waveguide by using standard coupled mode theory, i.e. \( \Psi \) is expanded in the eigenfunction basis \( \psi_\nu \) of the unperturbed waveguide (i.e. verifying \( \mathcal{H}\psi_\nu = \beta_\nu \psi_\nu \)) via \( \Psi(x,z) = \Psi^T(x)D_\beta(z)a(z) \) with \( \Psi^T = (\psi_0, \psi_1, \ldots)^T \). The diagonal matrix \( D_\beta \) contains the phase of each mode: \( (D_\beta)_{\nu \nu} = e^{-i\beta_\nu z} \) and \( a(z) \) contains the modes amplitudes. Now, by using orthogonality properties of guided modes, (4) is transformed into the system of ordinary differential equations

\[
\frac{da}{dz} = -i\gamma \left( e^{iz}A + e^{-iz}A^T \right) a, \tag{6}
\]

where the lower diagonal matrix \( A \) is the algebraic representation of the creation operator [2]. By exploiting the properties that \([A^T, A] = 1/2\) where \([\cdot, \cdot]\) denotes the matrix commutator, the solution can be recombined via (5) with the separable form \( G = \Psi^T(x) G(z) \Psi(x') \) where

\[
G(z) = e^{i\Theta(z)}D_\beta(z) \exp \left( -i\eta \Omega(z) \right), \tag{7}
\]

\( \Theta \) is a real-valued phase function and \( \Omega \) is the bidiagonal matrix \( \Omega(z) = g(z)A + g(z)A^T \) where \( g(z) = \int_0^z \partial_\sigma \varphi(\varepsilon z') e^{i\varepsilon z'} dz' \). Functions \( (G(z))_{\mu \nu} \) give the amplitude of mode \( \mu \) at \( z \) if only mode \( \nu \) is present at the waveguide input \( z = 0 \). Now, since \( \eta \sim \varepsilon \Delta^{-1} \sqrt{V} \), the scattering matrix \( G \) is expected to be fully populated even for low curvature and this renders the numerical evaluation of (7) very penalizing for waveguide supporting a very large number of modes.

3 Coupled bend mode theory

In order to take advantage of the weak dependence of the curvature with respect to the arc length, a better option is to treat \( \gamma \) as a fixed parameter and consider the eigenmode of the curved waveguide \( \psi_\nu^b = \psi_\nu^b(x;\gamma) \) satisfying \( (\mathcal{H} + \gamma x) \psi_\nu^b = \beta_\nu^b \psi_\nu^b \) where the eigenvalue now depends implicitly on the local radius of curvature, \( \beta_\nu^b = \beta_\nu^b(\gamma) \). By following the same analysis, we find the bend mode representation for the Green function: \( G = \Psi_0^b(x;\gamma) G_0(z) \Psi_0(x';\gamma) \) where \( G_0(z) = e^{i\Theta_0(z)}D_\beta(z) \exp \left( -i\eta \Omega_0(z) \right) \),

\[
\Theta_0_0^b(z) = \text{a real-valued phase function and } \Omega_0 \text{ is the bidiagonal matrix } \Omega_0(z) = g_0(z)A + g_0(z)A^T \text{ where } g_0(z) = \int_0^z \partial_\sigma \varphi(\varepsilon z') e^{i\varepsilon z'} dz'. \text{ Clearly the new decomposition (8) offers substantial improvements compared to the one associated with the straight mode scattering (7) since the numerical convergence of the exponential series is now controlled by the order of magnitude of } \epsilon \eta. \text{ Consider for instance an optical waveguide with } V = 100 \text{ (i.e. supporting approximately } V/2 = 50 \text{ modes), } \Delta = 0.01 \text{ curved with a typical radius of curvature of } 100a. \text{ We find that } \eta = 2.5 \text{ (making the series (7) poorly convergent), whereas } \epsilon \eta = 0.17. \text{ In this case, the first order expansion in (8) provides a good approximation. Numerical results, not shown here, confirm this analysis.}

References