

**Université de technologie de Compiègne - Proposition de thèse**

<b>1<sup>re</sup> partie : Fiche scientifique</b>	
Intitulé de la thèse	
Type de financement	Contrat doctoral sur allocation Ministère
Laboratoire d'accueil	unité de recherche : LMAC EA 2222 équipe de recherche : Systèmes Stochastiques site web : <a href="http://lmac.utc.fr">http://lmac.utc.fr</a>
Directeur(s) de thèse	Alaya Mokhtar et Salim Bouzebda
Domaines de compétence	Mathématiques Sciences pour l'ingénieur
Description du sujet de thèse	Voir ci après.
Mots clés	Convergence faible, localement stationnaire, processus empirique conditionnelle
Profil et compétences du candidat	Master 2 en Statistique, Probabilité, mathématique appliquée. Connaissance de Logiciels R ou Matlab.
Date de début de la thèse	01/10/2022
Lieu de travail de thèse	Laboratoire de mathématique appliquée de Compiègne

<b>2<sup>e</sup> partie : Fiche de poste</b>	
Durée	36 mois
Possibilité missions complémentaires	enseignement
Laboratoire d'accueil	LMAC : Modélisation stochastique, analyse numérique
Moyens matériels	bureau collectif, ordinateur, etc ; les moyens de l'unité : équipements utiles pour les travaux de thèse
Moyens humains	12 EC, 1 BIATSS/ITA, 10 doctorants
Moyens financiers	
Modalités de travail	autonomie attendue, fréquence de réunions avec les directeur de thèse : 1 à 2 réunions par mois (et plus si c'est nécessaire)
Projet de recherche lié à cette thèse	
Collaboration(s) nationale(s)	
Collaboration(s) internationale(s)	
Thèse en cotutelle internationale	Non
Coordonnées de la personne à contacter	<a href="mailto:elmokhtar.alaya@utc.fr">elmokhtar.alaya@utc.fr</a> <a href="mailto:Salim.bouzebda@utc.fr">Salim.bouzebda@utc.fr</a> , (+33) 3 44 23 44 69

**Contactez d'abord le directeur de thèse** avant de renseigner un dossier de candidature en ligne sur <https://webapplis.utc.fr/admissions/doctorants/accueil.isf>



# Conditional Empirical Processes for Locally Stationary Functional Time Series

PhD subject

Expected starting date: September/October 2022

**Key-words** Empirical Processes, Functional Time Series, Locally Stationary Processes

**Topic** Sciences de l'Ingénieur

**Context** In an increasing number of situations, the collected data appear as functional or curve time series coming from different research fields such as biometrics ([7]), environmetrics ([3]), econometrics ([4], [5]), and finance ([17], [6], [19]). We refer to [15] as standard references for functional time series analysis.

In the literature on functional time series analysis, most studies are based on (linear) stationary models (e.g., [11], [1], [3]). However, many functional time series exhibit a nonstationary behavior. For example, in the financial industry, implied volatility of an option as a function of moneyness changes over time. We can also find other examples of possibly nonstationary functional time series in [22]. One way to model nonstationary behavior is provided by the theory of locally stationary processes.

Locally stationary processes, as proposed by [8], are nonstationary time series that allow parameters of the time series to be time-dependent. They can be approximated by a stationary time series locally in time, which enables asymptotic theories to be established for the estimation of time-dependent characteristics. In time series analysis, locally stationary models are mainly considered in a parametric framework with time-varying coefficients. For example, we refer to [10], [13], [16] and [24]. Moreover, nonparametric methods for stationary and nonstationary time series models have also been developed. We refer to, among others, [20], [12] and [14] for stationary time series as well as [18], [23], and [21] for contributions on locally stationary time series. We refer to [9] for a general theory in the literature on locally stationary processes.

**Scientific objectives and expected achievements** The successful candidate will pursue a PhD project at the intersection of statistics, machine learning and optimal transport. We will first investigate the role of OT to deal with time series problem. A second step consists in developing a framework of conditional empirical processes for a locally stationary functional time series that takes values in a semi-metric space (e.g., Banach and Hilbert spaces). In particular, we first propose conditional empirical processes for a locally stationary functional time series with a regression function that is allowed to change smoothly over time. Then, we (1) derive the Glivenko-Cantelli results and (2) establish the Donsker theorems. To attain the first objective, we derive uniform convergence results for a class of empirical processes based on kernel averages, which are crucial for demonstrating our main results, which are of independent interest.

**Research environment/Location** The research will take place within the [LMAC laboratory](#) located at Compiègne, France. The PhD will be supervised by Salim Bouzebda and Mokhtar Z. Alaya (LMAC - UTC).

### Candidate profile

- Prerequisites: candidates are expected to be graduated in computer science and/or applied mathematics/statistics and/or machine learning and show an excellent academic profile.
- Work environment: the PhD thesis subject requires skills on software development tools, preferably Python language programming with machine learning packages.
- Candidates must have a good level on written English for the production of scientific papers in international conferences and journals.

**For more details** Feel free to contact by sending an email to Salim Bouzebda([salim.bouzebda@utc.fr](mailto:salim.bouzebda@utc.fr)) and Mokhtar Z. Alaya ([elmokhtar.alaya@utc.fr](mailto:elmokhtar.alaya@utc.fr)).

### References

- [1] Antoniadis, A., Paparoditis, E. and Sapatinas, T. (2006). A functional wavelet-kernel approach for time series prediction. *J. Roy. Statist. Soc. Ser. B* **86**, 837-857.
- [2] Aue, A. and van Delft, A. (2020). Testing for stationarity of functional time series in the frequency domain. *Ann. Statist.* **48**, 2505-2547.
- [3] Aue, A., Dubart Nourinho, D. and Hörmann, S. (2015). On the prediction of stationary functional time series. *J. Amer. Statist. Assoc.* **110**, 378-392.
- [4] Bugni, F., Hall, P., Horowitz, J. and Neumann G. (2009). Goodness-of-Fit Tests for Functional Data. *The Econometrics Journal* **12**, S1-S18.
- [5] Bugui, F. and Horowitz, J. (2018). Permutation tests for equality of distributions of functional data. arXiv:1803.00798.
- [6] Chen, S. X., Lei, L. and Tu, Y. (2016). Functional coefficient moving average model with applications to forecasting Chinese CPI. *Statistica Sinica* **26**, 1649-1672.
- [7] Chiou, J.-M. and Müller, H.-G. (2009). Modeling hazard rates as functional data for the analysis of cohort lifetables and mortality forecasting. *J. Amer. Statist. Assoc.* **104**, 572-585.
- [8] Dahlhaus, R. (1997). Fitting time series models to nonstationary processes. *Ann. Statist.* **25**, 1-37.
- [9] Dahlhaus, R., Richter, S. and Wu, W.B. (2019). Towards a general theory for nonlinear locally stationary processes. *Bernoulli* **25**, 1013-1044.
- [10] Dahlhaus, R. and Subba Rao, S. (2006). Statistical inference for time-varying ARCH processes. *Ann. Statist.* **34**, 1075-1114.
- [11] Dehling, H. and Sharipov, O. S. (2005). Estimation of mean and covariance operator for Banach space valued autoregressive processes with independent innovations. *Stat. Inference Stoch. Process.* **8**, 137-149.
- [12] Fan, J. and Yao, Q. (2003). *Nonlinear Time Series: Nonparametric and Parametric Methods*. Springer, New York.

- [13] Fryzlewicz, P., Sapatinas, T., and Subba Rao, S. (2008). Normalized least-squares estimation in time-varying ARCH models. *Ann. Statist.* **36**, 742-786.
- [14] Hansen, B. E. (2008). Uniform convergence rates for kernel estimation with dependent data. *Econometric Theory* **24**, 726-748.
- [15] Horváth, L. and Kokoszka, P. (2012). *Inference for Functional Data with Applications*. Springer, New York.
- [16] Koo, B. and Linton, O. (2012). Semiparametric estimation of locally stationary diffusion models. *J. Econometrics* **170**, 210-233.
- [17] Kokoszka, P. and Zhang, X. (2012). Functional prediction of intraday cumulative returns. *Statistical Modelling* **12**, 377-398.
- [18] Kristensen, D. (2009). Uniform convergence rates of kernel estimators with heterogeneous dependent data. *Econometric Theory* **25**, 1433-1445.
- [19] Li, D., Robinson, P. M. and Shang, H. L. (2020). Long-range dependent curve time series. *J. Amer. Statist. Assoc.* **115**, 957-971.
- [20] Masry, E. (1996). Multivariate local polynomial regression for time series: uniform strong consistency and rates. *J. Time Ser. Anal.* **17**, 571-599.
- [21] Truquet, L. (2019). Local stationarity and time-inhomogeneous Markov chains. *Ann. Statist.* **47**, 2023-2050.
- [22] van Delft, A. and Eichler, M. (2018). Locally stationary functional time series. *Electron. J. Statist.* **12**, 107-170.
- [23] Zhang, T. and Wu, W. B. (2015). Time-varying nonlinear regression models: Nonparametric estimation and model selection. *Ann. Statist.* **43**, 741-768.
- [24] Zhou, Z. (2014). Nonparametric specification for non-stationary time series regression. *Bernoulli* **20**, 78-108.