A Roadmap Matching Method for Precise Vehicle Localization using Belief Theory and Kalman Filtering

M.E. EL Najjar and Ph. Bonnifait
Heudiasyc UMR 6599
Université de Technologie de Compiègne.
BP 20529, 60205 Compiègne Cedex, France.
maan.el-najjar@hds.utc.fr

Abstract
A novel roadmap matching method is developed. This method provides an accurate position of a vehicle relatively to a digital road map using Belief Theory and Kalman filtering. Firstly, an Extended Kalman Filter fuses DGPS and ABS sensors measurements to estimate a rough pose of the vehicle. This pose is then used to select the most likely segment from the database. The strategy fuses several criteria based on distance, heading and velocity measurements using Belief Theory. Finally, a new observation is built thanks to the selected segment, and used to correct the rough pose thanks to a second Kalman filter estimation stage. A special attention has been given to the modeling of the system. It appeared that augmenting the state with the bias of the roadmap significantly increases the performance of the method. Real experimental results show that this method is able to work a long time without GPS, if it has been correctly initialized.

1. Introduction
Many modern in-vehicle navigation and safety applications require real-time positioning of the vehicle with respect to a given set of digital map data. As a matter of fact, real-time positioning allows the driving assistance module to accurately depict the position of the vehicle on the map, facilitates operations such as route calculation, supports Advanced Driver Assistance System applications - ADAS - such as Adaptive Cruise Control - ACC, adaptive lighting control, collision warning, lane departure warning, etc. The quality of the localization process depends mainly on the quality of the road map matching which is a complicated problem when one seeks to obtain a reliable, precise and robust vehicle location on the road network [2, 8].

Today, the positioning system relies very often on the use of GPS, because it is an affordable and convenient sensor. Nevertheless, GPS suffers from satellite masks occurring in urban environments, under bridges, tunnels or in forests. GPS appears then as an intermittent positioning system that demands the help of a dead-reckoning system [1]. In this paper, we use the rear wheels ABS sensors of the car for this purpose. This multisensor fusion is performed by an Extended Kalman Filter (EKF).

The selection of candidate roads is the first stage of the roadmap-matching problem [7]. Generally, it consists in applying a first filter which selects all the segments close to the estimated position of the vehicle. Then, the goal is to select the most likely segment(s) from this subset. Nowadays, since the geometry of the roadmaps is more and more detailed, the number of segments that represent the roads is increasing. The road selection is therefore essential for the stability of the system and because, if this stage is neglected, the computation time of many roadmap-matching algorithms cannot be acceptable for a real-time implementation. In order to be focused on this point, an accurate map Géoroute V2 provided by the French National Institute of Geography (IGN) has been used in this work. Our strategy is based on the fusion of several criteria (using distance, heading and velocity measurements, connectivity with the latest matched road and one way restrictions) using Belief theory.

A more accurate location of the vehicle can be obtained by fusing the selected segment with the odometry / GPS estimated pose. The key idea is to model the fact that the true position of the vehicle is located around the centerline of the most likely road. This region depends mainly on the width of the road, which is an information, stored in the digital map database as an ADAS attribute. So, the most likely road is considered like a new Kalman observation and its associated error is estimated. The map observation modeling is an important contribution of this paper.

The outline is as follows. In section 2, the architecture of the roadmap-matching method is described. The state space formulation of the problem, the road selection, the map observation and the EKF are detailed. In section 3, real data results are analyzed. Finally, the concluding remarks are made in section 4.
2. Description of the roadmap matching method

The roadmap matching method described in this section relies on Kalman filtering like in [6]. We suppose that the reader is familiarized with this formalism, so, we will just detail the state space representation, i.e. the state vector, the evolution model, the observation model and the covariances of the errors.

2.1. Evolution model: odometry

Let consider a car like vehicle with front-wheel drive. The mobile frame is chosen with its origin M attached to the center of the rear axle. The x-axis is aligned with the longitudinal axis of the car (see Fig 1).

![Fig. 1. The mobile frame attached to the car](image)

The vehicle position is represented by the \((x_k, y_k)\) Cartesian coordinates of M in a world frame. The heading angle is denoted \(\theta_k\). If the road is perfectly planar and horizontal and, if the motion is locally circular, the evolution model can be expressed by [4]:

\[
\begin{align*}
x_{k+1} &= x_k + \delta_s \cos(\theta_k + \delta_\theta / 2) \\
y_{k+1} &= y_k + \delta_s \sin(\theta_k + \delta_\theta / 2) \\
\theta_{k+1} &= \theta_k + \delta_\theta
\end{align*}
\]

(1)

where \(\delta_s\) is the length of the circular arc followed by M, \(\delta_\theta\) the elementary rotation of the mobile frame. These values are computed using the ABS measurements of the rear wheels.

2.2. Observation equation

When a GPS position is available, a correction of the odometric estimation is performed using an Extended Kalman Filter (EKF). If the GPS satellites signal is blocked by buildings or tunnels, for example, the evolution model provides a dead-reckoned pose. This rough pose is used to select the most likely segment(s) from the database. This approach is presented in section 2.3. If several segments are candidates, the observation function is non-linear.

\[
Y = f(X_k) + \beta_k
\]

(2)

Two main strategies can deal with this non-linearity:
- the management of multi-hypotheses
- the selection of the most likely segment among the segments set.

In this paper, we consider the second solution. The most likely segment is used to construct a map observation, denoted \((x_h, y_h)\), and its associated error (see section 2.4). Therefore, the observation equation becomes linear:

\[
\begin{bmatrix}
x_{gps} \\ y_{gps} \\ x_h \\ y_h
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x \\ y \\ \theta
\end{bmatrix} + \beta_k
\]

(3)

\(\beta_k\) represents the observation error.

The GPS measurement error can be estimated in real time using the NMEA sentence "GST" provided by the Trimble AgGPS132 receiver which has been used in the experiments. Therefore, the GPS noise is not stationary.

If we assume that the GPS position and the map observation errors are not correlated, the covariance matrix of the complete measurement \(Y\) can be separated in two blocs:
- \(Q_{gps}\): covariance matrix of the GPS error
- \(Q_h\): covariance matrix of the map observation error.

\[
Q_k =
\begin{bmatrix}
\sigma_{x,gps}^2 & 0 & 0 & 0 \\ 0 & \sigma_{y,gps}^2 & 0 & 0 \\ 0 & 0 & \sigma_{x,h}^2 & 0 \\ 0 & 0 & 0 & \sigma_{y,h}^2
\end{bmatrix}
\]

(4)

Since \(Q_k\) is diagonal, the GPS and map observations can be used in two separated Kalman filter estimation stages.

2.3. Segment selection using Belief Theory

Belief theory allows one to reason with uncertainty and suggests a way for combining uncertain data. This theory is the generalization of Bayes theory in the treatment of uncertainty. Generally, this theory is used in a multi-sensor context to fuse heterogeneous information in order to obtain the best decision.

In the first place, a frame of discernment has to be defined. It is the set of all possible answers to a specific question. In our particular case, the segments are treated...
one by one and a pertinent question is: "is this segment the good one?" The response can be: yes, no, or perhaps. The frame of discernment is then $\Theta=\{\text{Yes, No}\}$ and the mass assignments are computed on the definition referential $2^\Theta=\{\text{Yes, No, Perhaps}\}$. The mass assignments allocate a degree of Belief to each hypothesis. A mass assignment must satisfy the following rules:

$$m(\emptyset) = 0 \sum_{A\subseteq\Theta} m(A) = 1 \quad (5)$$

Each mass function $m(.)$ is represented by a real number in $[0,1]$. The mass notion is very near to the probabilistic mass notion, exception that it is not shared only among single hypotheses but it is also possible to attribute a mass to an union of hypotheses.

In practice, the selection criteria must be clearly identified in order to assign masses. The criteria used in this article can be formulated as follows:

- the vehicle location is close to a segment of the neighborhood. This criterion is a function of the error ellipse (on the assumption that the error distribution is Gaussian),
- the segments corresponding to the vehicle location are those which have an angle close to the heading of the vehicle. This criterion a function of the estimated $\sigma$ bound of the heading and a function of the speed,
- if at time $k$, an arc is selected without any ambiguity, at time $k+1$, the good segment would belong to this arc or to an arc having an extremity connected to it,
- the vehicle orientation should respect the traffic direction of the road (in case of a one-way road). This ADAS attribute is stored in the map database.

In order to assign masses, criteria have to be fuzzified. For more information on that point one can refer to [5]. Then the mass functions are fused and a decision has to be taken. Several decision laws can be used. In this work, we have used the "ideal decision law" in order to keep only the non ambiguous segments.

Let consider a concrete case to illustrate the method with the proximity and heading criteria only. In figure 2, the vehicle is running on the road represented by the segments 1 and 3, at a speed of 80 km/h. Estimation errors and roadmap errors induce an erroneous estimated position, which is closer to segment 2 than to the others. The mass assignments for each segment using the two criteria are indicated on figure 2.

One can remark that for segment 1, the proximity criterion and the heading criterion are in accordance because both of them assigns a large Belief to the Yes hypothesis, a little to the Perhaps hypothesis and nothing to the No hypothesis. Conversely, segment 2 presents a total conflict between the two criteria. With these mass assignments, the fusion will indicate that only segments 1 and 3 respect the ideal decision law.

Note that the key characteristic of this road selection strategy is the quantification of the Belief in the selected segments. As we have chosen a very reliable decision strategy, one can be sure that, when the selection stage provides a segment, this segment corresponds to the real road with a high probability. On the contrary, if there is no credible segment, this method indicates that the car is off-road or on a road which is not stored on the database.

### 2.4. Map observation

A way to fuse the most likely segment with the other sensors is to treat it like an observation, that is a function of the state vector. Much effort has been spent on modeling the map observation error in a realistic way. It has turned out that a Gaussian mixture which encloses the road works well.

To build the map observation, we consider two cases.

**Principle: case of a straight road**

![Fig. 2. Estimated pose of the vehicle and 3 candidate segments.](image)

![Fig. 3. Case of a straight road.](image)
The simplest case occurs when the most likely segment corresponds to a straight road. In figure 3, the road selection stage provides three segments (in bold). Their masses of Belief are 0.9, 0.7 and 0.55 for segments 1, 2 and 3, respectively. As all these segments are credible (decision given by the road selection stage), it is an ambiguous situation and the map observation is non linear. A way to circumvent this difficulty is to select the segment which has the highest Belief value (segment 1 here). The map observation \( \xi_h, \eta_h \) is defined as the orthogonal projection of the estimated position \((x_h, y_h)\) onto segment 1 [3]. The map observation can be constructed in a different way. In [6], the observation is the distance to the nearest road. In the Recursive Bayesian Estimation context (i.e. a particle filter), this (non linear) measurement should be equal to zero.

The problem is now to estimate the error of this observation.

The box surrounding the segment and representing the road determines the maximum error of the map observation. The resulting probability density function is then theoretically spatially truncated. In the Kalman filtering context, this box is approximated by a Gaussian ellipse as shown on figures 3 and 4.

Let consider a local frame attached to the segment. Its axis is collinear to the segment. In this frame, the Gaussian ellipse is oriented along the road segment and the coordinates of its center are \((x_h, y_h)\).

The error is then given by:

\[
Q^E_h = \begin{bmatrix}
\sigma^E_{xh} & 0 \\
0 & \sigma^E_{yh}
\end{bmatrix}
\]

where \(\sigma^E_{xh}\) and \(\sigma^E_{yh}\) are the longitudinal and the transversal standard deviations. If the segment is infinite, then \(\sigma^E_{xh} = \infty\). This means that the map measurement can only correct the position in the \(y\) direction. The transversal standard deviation is given by:

\[
\sigma^E_{yh} = \left(\frac{w_s}{2k}\right)
\]

where \(w_s\) is the width of the road segment (\(w_s\) is stored in the database as an ADAS attribute) and \(k\) is the constant associated with the chosen probability error ellipse \(P\) given by:

\[
k = \sqrt{-2\ln(1-P)}
\]

**Case of a curved road**

In general, the roads are not straight and, because the new digital roadmaps include many details, the segments have a small length (see Fig. 4).

The map observation is the nearest point from \((x_h, y_h)\) making part of the most likely segment. Depending on the case, it is the orthogonal projection of \((x_h, y_h)\) onto the segment or one of its extremities. One can notice that the most likely segment represents a linearization of the curved road.

In this case, \(\sigma^E_{xh}\) is not infinite, but it has to be chosen big enough in comparison with \(\sigma^E_{yh}\) to indicate that the correction is higher in the \(y\) direction (in the local frame attached to the segment).

In the reference frame, the covariance matrix \(Q_h\) of the map observation is obtained from \(Q^E_h\) (see equation (6)) by a simple rotation. If \(\alpha\) is the orientation of the segment with respect to the \(x\) axis of the reference frame, then:

\[
Q_h = \begin{bmatrix}
\sigma^2_{xh} & \sigma^2_{xh} \\
\sigma^2_{xh} & \sigma^2_{yh}
\end{bmatrix}
\]

where

\[
\sigma^2_{xh} = (\sigma^E_{xh})^2 \cos^2(\alpha) + (\sigma^E_{yh})^2 \sin^2(\alpha)
\]

\[
\sigma^2_{yh} = (\sigma^E_{xh})^2 \sin^2(\alpha) + (\sigma^E_{yh})^2 \cos^2(\alpha)
\]

\[
\sigma^2_{xh} = ((\sigma^E_{xh})^2 - (\sigma^E_{yh})^2) \cos(\alpha) \sin(\alpha)
\]

**2.5. Augmenting the state vector**

The roads are symbolized by arcs whereas the car is running on a surface centered on these arcs. Moreover, the geometrical transformation between the GPS reference frame and the French Lambert projection frame is performed using the following geometrical transformation:

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
\cos(\alpha) & \sin(\alpha) \\
-\sin(\alpha) & \cos(\alpha)
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}
\]

where \(\alpha\) is the orientation of the segment. The resulting position \((x', y')\) is the estimated position \((x_h, y_h)\).
frame can present an offset of several meters (~5m). Finally, the segments coordinates present errors because of the plotting of the geographers and because the coordinates are stored as integers in the database (1 m roundness).

For all these reasons, the fusion of the GPS with odometry has a variable offset with the map data. A solution to this problem is to add two offsets (denoted $d_x$ and $d_y$) in the state vector and to observe them.

As the goal of the positioning module is to localize the car on the road network (because the ADAS attributes are attached to this network), it is the GPS measurement which presents an offset rather than the map observation.

Let denote $X=[x, y, \theta, \delta_x, \delta_y]^T$ and $U=[d_x, d_y]^T$. By rewriting Eq.11 and 12, we obtain the state space representation:

$$
\begin{align*}
X_{k+1} &= f(X_k, U_k) + \alpha_k \\
y_k &= C X_k + \beta_k
\end{align*}
$$

The observation model is linear whereas the evolution one is non linear. An Extended Kalman Filter with a measured input is then a way to fuse all this data.

### 2.6. Overview of the roadmap matching method

The algorithm firstly fuses the ABS measurements with a DGPS position, if it is available. Then, using this estimation, the credible roads are selected. If at least one segment is credible, the map observation is built and fused with the other data thanks to a second Kalman filter estimation stage.

3. Experimental results

Figure 6 presents a top view of a 4-km long experimental test performed in Compiègne. The map data-base was managed by the GIS software "Geoncept". The DGPS receiver used was a Trimble AgGPS132 with an Omnistar differential correction.

In the following, the map observation covariance matrix has been chosen with $\sigma_{sh}^2 = 150$ m and $P=0.9$.

In figure 6, the gray path corresponds to the rough absolute positions provided by the DGPS transformed in the French Lambert coordinates system of the map. The black path is the result of the fusion of the sensors with the roadmap. In this experience, a DGPS signal mask has been simulated (i.e. the DGPS measurements haven’t been used). This signal mask starts at the exit of the first roundabout (in the bottom of Fig. 6). One can remark that in spite of the long DGPS mask (about two kilometers), the vehicle location is matched correctly. As a matter of fact, the final estimated positions stay close to the DGPS points.
The fine performance of the road selection method is illustrated on figure 7. Between the two roundabouts, the journey is a 2x2-lane road: each roadway is represented by a one-way arc. In spite of the closeness of the DGPS positions to the wrong arc, the estimated poses are associated to the good arc. Moreover, the roundabouts correct efficaciously the estimation even if they are represented by many segments which lead to ambiguous situations.

In order to prove the interest of the map observation, figure 8 shows the dead-reckoning results using the ABS sensors only, when the GPS signal is not available. One can see that the map observation corrects efficiently the drift of odometry (the GPS mask still starts at the exit of the roundabout in the bottom of the figure).

4. Conclusion

This article has presented a roadmap matching method based on multi-sensor fusion and on multi-criteria fusion for the road selection. The main contributions of this work are the formalization of a roadmap observation in the Kalman filtering context, the use of a road selection method based on multi-criteria fusion using Belief theory, and an experimental validation with real data.

It seems that the GPS measurements are not necessary all the time since the fusion of odometry and roadmap data can provide a good estimation of the position for a long time. Nevertheless, we have noticed that sometimes this estimation can diverge. This is due to the fact that the strategy presented in this paper keeps only the most likely segment. When approaching an intersection, several roads can be good candidates. If a wrong road is more credible than the good one, the method will diverge, because the GPS is not available to correct this wrong choice. A solution to this problem is to manage several hypotheses until the situation becomes unambiguous. We think that is the main perspective of this research.

5. References